
If there is a problem you can't solve, then there is an easier problem you can solve: find it.
– George Polya

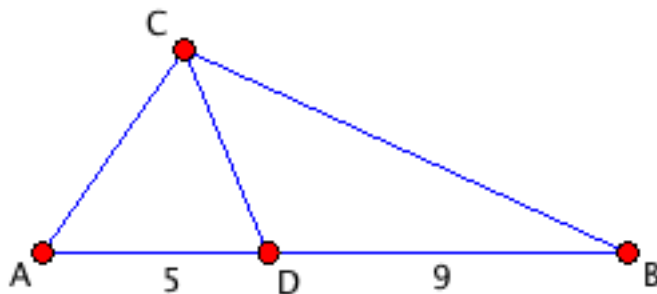
NYU High School B

Instructor: David Gomprecht

The NYMC High School B class covers challenging topics at the 10th grade level. We do not assume that students know Precalculus or Calculus. Topics include algebra, geometry, number theory, combinatorics, probability, induction, recursion, polynomials, and inequalities. The problems can be difficult, but hopefully they're also interesting and fun. Here are some sample problems.

1. Problems that you ought to be able to solve before taking this class

- (a) What is the value of $\frac{[DBC]}{[ADC]}$? (The notation $[ABC]$ means the area of $\triangle ABC$.)



- (b) How many positive whole numbers less than or equal to 1,200 are multiples of 2 or 3?
- (c) How many 5 digit numbers can be formed using each of the digits 0, 2, 4, 6, and 8 exactly once?
- (d) The quadratic polynomial $x^2 + 2kx + 3k$ has exactly one root. What are the possible values of k ?

2. Examples of interesting problems that we'll consider in this class

- (a) How many zeros are at the end of $1000!$?
- (b) 9 girls and 3 boys sit in one row of a classroom. How many seating arrangements are there, if none of the boys are allowed to sit next to each other?
- (c) Find the smallest n such that the n^{th} Fibonacci number is divisible by 30.
- (d) The polynomial $P(x)$ has remainder 3 when divided by $x - 1$ and remainder 5 when divided by $x + 1$. What is the remainder when $P(x)$ is divided by $x^2 - 1$?
- (e) A gecko and an anti-gecko take a random walk on the faces of a cube, starting on opposite faces. How long, on average, will it be before the geckos walk onto the same face of the cube and annihilate each other?
- (f) A penguin takes a random walk on a long ladder, starting at the intersection of one of the rungs and one of the sides. Every second she moves, with equal probability, either to the other side of the ladder (on the same rung) or up or down one rung (on the same side). However, she only moves up on one side of the ladder, and she only moves down on the other side. Find the probability that, after one minute, the penguin is back where she started.
- (g) Given a circle and two points on the circle, construct a pair of parallel chords, one chord through each of the given points, such that the sum of the lengths of the chords is equal to the length of the given segment.
- (h) Prove Nesbitt's inequality, for a , b , and c non-negative:

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$$