HS A

1. c
2. d
3. b
4. a
5. b
6. d
7. d
8. d
9. e
10. x = 3
11. $3x^3(x+4)^2$
12. $\frac{2}{7(3x=1)}$, $x \neq 0$, $x \neq -3$, $x \neq \frac{1}{2}$
13. 9 $(4! - 1 - 6 - 8)$
14. $a = 5, b = 8, c = -2, d = 7, e = -1$
15. 4034
16. 1008
17. 37037037
18. 3.25
19. (a) f(5)=16

    (b) k=63

    (c) $\frac{n(n+1)}{2} + 1$

HS B

1. d
2. c
3. a
4. a
5. b
6. d
7. c
8. d
9. a
10. $\frac{2}{7(3x=1)}$, $x \neq 0$, $x \neq -3$, $x \neq \frac{1}{2}$
11. 17
12. 4π
13. 93
14. 100π
15. 58
16. 40
17. (a) m+n-1

    (b) 3

    (c) $\frac{2m+n-1}{n+1}$

    (d) 9
18. (a) $s_5 = \{0, 1, 1, 2, 1, 3, 2, 3, 1, 4, 3, 5, 2, 5, 3, 4, 1\}$

    (b) $2^n + 1$

    (c) By induction. Need to show that $\Sigma_n = \frac{3^{n-1}+1}{2}$. Base case $n = 1$ is true.
    Assume $\Sigma_k = \frac{3^{k-1}+1}{2}$ for some k. Let $S_k = \{a_1, a_2, \cdots, a_{n_k}\}$ where $n_k = 2^k + 1$.
    Then $S_{k+1} = \{a_1, a_1 + a_2, a_2, a_2 + a_3, \cdots, a_{n_k-1}, a_{n_k-1} + a_{n_k}, a_{n_k}\}$
    So $\Sigma_{k+1} = 3\Sigma_k - a_1 - a_{n_k} = 3\Sigma_k - 1 = 3\frac{3^{k-1}+1}{2} - 1 = \frac{3^k+1}{2}$ QED

HS C

1. c
2. b
3. a
4. c
5. b
6. e
7. c
8. b
9. a
10. $\frac{2}{7(3x=1)}$, $x \neq 0$, $x \neq -3$, $x \neq \frac{1}{2}$
11. x = -3
12. 9876543120
13. 4
14. 502
15. 3.2
16. 7
17. (a) $s_5 = \{0, 1, 1, 2, 1, 3, 2, 3, 1, 4, 3, 5, 2, 5, 3, 4, 1\}$

    (b) $2^n + 1$

    (c) We will prove that S_n contains F_n and F_{n-1} next to each other, by induction.
    S_2 contains F_1 and F_2 next to each other.
    Assume that S_k contains F_k and F_{k-1} next to each other.
    It follows that S_{k+1} contains the sequence $F_{k-1}, F_{k-1} + F_k, F_k$
    However, $F_{k-1} + F_k = F_{k+1}$
    So F_{k+1} and F_k are adjacent QED.
    It follows that S_n contains F_n.

    (d) By induction. Need to show that $\Sigma_n = \frac{3^{n-1}+1}{2}$. Base case $n = 1$ is true.
    Assume $\Sigma_k = \frac{3^{k-1}+1}{2}$ for some k. Let $S_k = \{a_1, a_2, \cdots, a_{n_k}\}$ where $n_k = 2^k + 1$.
    Then $S_{k+1} = \{a_1, a_1 + a_2, a_2, a_2 + a_3, \cdots, a_{n_k - 1}, a_{n_k - 1} + a_{n_k}, a_{n_k}\}$
    So $\Sigma_{k+1} = 3\Sigma_k - a_1 - a_{n_k} = 3\Sigma_k - 1 = 3\frac{3^{k-1}+1}{2} - 1 = \frac{3^k+1}{2}$ QED