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1. Find five different positive integers, two of which are less than 100, such that each has exactly 12 distinct positive integer factors.
  2. For the integer  $(2^{10})(3^5)$ , compute the number of
    - (a) even factors
    - (b) perfect square factors
    - (c) perfect cube factors
  3. Find the smallest positive integer  $k$  such that 990 divides  $k!$
  4. How many multiples of 7 or 11 are there between 1 and 1000?
  5. Find the remainder when  $7^{105}$  is divided by 10.
  6. A Pythagorean Right Triangle (PRT) is a right triangle all of whose sides have positive integer lengths. Find all PRT's having a hypotenuse of 65.
  7. Ten fair coins are tossed once. What is the probability of getting precisely 4 heads and 6 tails?
  8. How many arrangements are there of the letters of the word ORANGE such that
    - (a) the letters O and R must be together?
    - (b) the letters O and R may not be together?
  9. How many positive integers  $N$ , where  $1 \leq N \leq 1000$ , do not contain the digit 7?
  10. How many solutions  $(x, y, z)$  are there of  $x + y + z = 10$ , where  $x, y$  and  $z$  are positive integers? Note:  $(1, 2, 7)$  and  $(2, 1, 7)$ , for example, represent different solutions.
  11. Show that the repunit  $111 \cdots 11$  containing exactly 91 1's is composite.
  12. Compute the largest prime factor of  $3^{14} + 3^{13} - 12$