Multiple Choice: Indicate your answer in the box to the right of each question.

1. Evaluate $(x-1)(x^2-2)(x^3-3)$ when $x=-2$

(a) 10 (b) -22 (c) -30 (d) 66 (e) -198

1.

---

2. If the roots of $y=2x^2-7x-5$ are s and t, compute the value of $(s+1)(t+1)$

(a) 1 (b) $\sqrt{89}-7$ (c) 2 (d) $\frac{\sqrt{89}-5}{2}$ (e) $11-\sqrt{89}$

2.

---

3. If $\cos 2x = 5\cos x - 3$, then a possible value of $\cos x$ is:

(a) $-\frac{1}{2}$ (b) $-\frac{2}{5}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$ (e) $\frac{4}{5}$

3.

---

4. The equation $4^{x+1}=13(2^x)-3$ has two real solutions, s and t. Find $s+t$.

(a) $\log_2\frac{3}{4}$ (b) $-2+\log_4 3$ (c) $\frac{13}{4}$ (d) -13 (e) $\log_2 13$

4.

---

5. For some integer n, the non-zero digits of $n!$ are 1, 2, 2, 3, 4, 4, 6, 6, 7, 8, 9, and X. Find X.

(a) 1 (b) 2 (c) 4 (d) 5 (e) 9

5.

---

6. The sum of all but one of the interior angles of a convex polygon equals 2016°. The remaining angle measures:

(a) 36° (b) 54° (c) 72° (d) 144° (e) Cannot be determined

6.

---

7. When 17! is written in base 12, how many zeroes does it end in?

(a) 1 (b) 4 (c) 6 (d) 7 (e) 9

7.

---

8. If $\tan A$, $\tan(A+B)$, and $\tan B$ form an arithmetic sequence (in that order), find $\tan A \tan B$.

(a) -1 (b) 0 (c) $\frac{\sqrt{2}}{2}$ (d) $\sqrt{2}$ (e) 2

8.

---

9. Right triangles ABC and ABD overlap. If $AB=65$, $AC=52$, $BC=39$, $AD=60$, and $BD=25$, find the length CD.

(a) 6 (b) 13 (c) 15 (d) 16 (e) 18

9.

Short Answer: Write your answer and show your work in the space below each question.
Clearly indicate your final answer by drawing a box around it.

10. Solve for x: $\frac{3x^2}{x+2} = \frac{x^2-x}{x^2+x-2}$

11. Evaluate $(\log_2 3)(\log_3 4)(\log_4 5) \ldots (\log_{2015} 2016)$

12. Given that $x > y$, solve:

$$x + 2xy + y = 11$$
$$x^2 + 6xy + y^2 = 57$$

13. If all the **even** positive integral factors of 2016 are written out, what is the median of those numbers?

14. Let A be the sum of the digits of the 31-digit number 1001^{10}. Given that (surprisingly) $A < 50$, find the sum of the digits of A.

15. How many of the first 2016 numbers are multiples of 6, 14, or 21, but not all three?

16. An isosceles trapezoid has sides 13, 23, 13, and 33. Compute the tangent of the acute angle made by its diagonals.

Long Answer: Write your solution in the space below each question. Make sure you include sufficient justification.

17. For some positive integers n, the expression $2^n + 105$ is a perfect square.

a. Show that n cannot be odd.
b. Assuming that n has to be even, find all values of n for which $2^n + 105$ is a perfect square.

18. (Langford's Problem) A Langford sequence of size n is a sequence of $2n$ numbers in which each number from 1 to n occurs twice, and the two occurrences of each number i have exactly i numbers between them. For example, 231213 is a Langford sequence of size 3. (Not that the two 1's have exactly 1 number between them, etc.)

a. Find a Langford sequence of size 4.
b. Find a Langford sequence of size 5, or prove that it does not exist.
c. What is the smallest $n > 5$ for which a Langford sequence of size n exists? Can you find one?