

**Multiple Choice:** Indicate your answer in the box to the right of each question.

1. Evaluate  $(x - 1)(x^2 - 2)(x^3 - 3)$  when  $x = -2$

- (a) 10      (b) -22      (c) -30      (d) 66      (e) -198

1.

2. If  $x + y = \frac{3}{5}$ ,  $y + z = \frac{4}{5}$ , and  $x + z = \frac{1}{3}$ , what is the value of  $x + y - z$ ?

- (a)  $\frac{1}{3}$       (b)  $\frac{2}{5}$       (c)  $\frac{3}{5}$       (d)  $\frac{2}{3}$       (e)  $\frac{4}{5}$

2.

3. What is the day and time that occurs 100 hours after 2 p.m. on Monday?

- (a) Fri. 6 p.m.    (b) Sat. 6 a.m.    (c) Sat. 8 a.m.    (d) Sun. 4 a.m.    (e) Sun. 8 p.m.

3.

4. The sequence 3, 6, 11, 22, 27, ... is formed by starting with 3 and alternately doubling and adding 5 to the previous number. Which of the following numbers eventually occurs in this sequence?

- (a) 1016      (b) 1017      (c) 1018      (d) 1019      (e) None of these

4.

5. How many distinct positive integral factors does  $(3!)(4!)$  have?

- (a) 6      (b) 11      (c) 13      (d) 15      (e) 16

5.

6. If the roots of  $y = 2x^2 - 7x - 5$  are  $s$  and  $t$ , compute the value of  $(s + 1)(t + 1)$

- (a) 1      (b)  $\sqrt{89} - 7$       (c) 2      (d)  $\frac{\sqrt{89}-5}{2}$       (e)  $11 - \sqrt{89}$

6.

7. Find the sum of the solutions of  $||x - 1| - 2| = 2016$

- (a) 0      (b) 2      (c) 4      (d) 4032      (e) 8096

7.

8. The parabola  $y = x^2 + ax + b$  has vertex  $(1, 7)$ . Find  $a$ .

- (a) -7      (b) -2      (c) 0      (d) 1      (e) 14

8.

9. When the numbers 1 through 2016 are written out, how many times will the digit 2 occur on the list?

- (a) 615      (b) 616      (c) 617      (d) 618      (e) 619

9.

**Short Answer:** Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a **box** around it.

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10. Write  $0.20161616\dots$  ("16" repeated) as a proper fraction in lowest terms.

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11. Solve for  $x$ :  $\frac{3x^2}{x+2} = \frac{x^2-x}{x^2+x-2}$

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12. If all the positive integral factors of 2016 are written out, what is the median of those numbers?

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13. For some positive integers  $a$  and  $b$ ,  $\frac{a+b^{-1}}{a^{-1}+b} = 17$ . If  $a$  is a prime number, find  $b$ .

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14. A triangular lake has sides of 3, 4, and 5 miles. Two friends start at the midpoint of the longest side and walk the perimeter of the lake in opposite directions with same speeds. What is the straight line distance between their starting point and where they meet again?

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15. Given that  $x > y$ , solve:

$$x + 2xy + y = 11$$

$$x^2 + 6xy + y^2 = 57$$

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16. The center of a circle with area  $200\pi$  lies on the circumference of a circle with area  $100\pi$ . Find the area of the region the two circles have in common.

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**Long Answer:** Write your solution in the space below each question. Make sure you include sufficient justification.

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17. Define a **current** number to be a natural number that ends in 6 and whose digits are alternating 1 and 6. For example, 6, 16, 616, 1616, and 61616 are all current numbers.

- a. Find a current number divisible by 9.
- b. Find the smallest current number divisible by 9. You should explain why it's the smallest.
- c. Can a current number be divisible by 19? Justify your answer.
- d. Which number(s) aren't factors of any current numbers?

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18. (Langford's Problem) A Langford sequence of size  $n$  is a sequence of  $2n$  numbers in which each number from 1 to  $n$  occurs twice, and the two occurrences of each number  $i$  have exactly  $i$  numbers between them. For example, 231213 is a Langford sequence of size 3. (Not that the two 1's have exactly 1 number between them, etc.)

- a. Find a Langford sequence of size 4.
- b. Find a Langford sequence of size 5, or prove that it does not exist.