

Multiple Choice: Indicate your answer in the box to the right of each question.

1. Evaluate $-3x^3 - (20 - x^4)$ when $x = -2$
 (a) 20 (b) -10 (c) 28 (d) -12 (e) None of these

1.

2. The coefficient of ab^3c in $(a + b + c)^5$
 (a) 15 (b) 20 (c) 25 (d) 30 (e) None of these

2.

3. Solve for A : $4^A = 2^{A+3} + 9$
 (a) 0 (b) $\log_2 3$ (c) $\log_4 3$ (d) $\log_2 9$ (e) None of these

3.

4. If $\sin x + \cos x = \frac{\sqrt{5}}{2}$ then $\sin 2x$ equals
 (a) $\frac{1}{4}$ (b) $-\frac{\sqrt{5}}{4}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$ (e) None of these

4.

5. Compute the tens digit of $1! + 2! + 3! + \dots + 2015!$
 (a) 1 (b) 3 (c) 9 (d) 0 (e) None of these

5.

6. How many distinct positive factors does $2^4 3^3 4^2$ have?
 (a) 21 (b) 27 (c) 45 (d) 60 (e) None of these

6.

7. If the roots of $y = x^2 - 5x + 2$ are s and t , compute the value of $\frac{s}{t} + \frac{t}{s}$.
 (a) $-\frac{29}{10}$ (b) $\frac{20}{7}$ (c) $\frac{21}{2}$ (d) 7 (e) None of these

7.

8. If $2 \cos x$, $3 \sin x$, $4 \tan x$ form a geometric sequence (in that order), find $\sin x$
 (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{8}{9}$ (e) None of these

8.

9. In a circle of radius r a chord of length 24 splits the perpendicular diameter into segments whose lengths are in a ratio of 1:9. Compute r .
 (a) $8\sqrt{13}$ (b) 20 (c) 15 (d) $10\sqrt{10}$ (e) None of these

9.

Short Answer: Write your answer and show your work in the space below each question.

Clearly indicate your final answer by drawing a box around it.

10. Simplify the expression: $\frac{x^2-5x+6}{x^3+4x^2} \cdot \frac{2x}{x-3} \cdot \frac{x^2+6x+8}{x^2-4}$

11. If $x + y = 4$ and $x^2 + y^2 = 10$, compute the value of $x^3 + y^3$.

12. How many of the integers from 1 to 2015, inclusive, can be represented as the difference of two squares?

13. What is the smallest positive power of x that does NOT appear in the expansion of $(1 + x + x^3)^{10}$?

14. Compute the remainder when $x^{2015} + 1$ is divided by $x^2 - 1$.

15. $\tan A = \frac{\cos 2015^\circ + \sin 2015^\circ}{\cos 2015^\circ - \sin 2015^\circ}$ and $0^\circ \leq A < 180^\circ$. Compute A .

16. A quadrilateral with sides 8, 13, 13, 18 is inscribed in a circle. The two congruent sides are adjacent. Find its area.

Long Answer: Write your solution in the space below each question. Make sure you include sufficient justification.

17. Prove that the equation $x^2 = 3 + 7y^2$ has no solutions in integers.

18. The Fibonacci numbers are defined as $F_1 = F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. Show that there exists an integer A such that the sum of any ten consecutive Fibonacci numbers can be represented as A times some Fibonacci number (possibly from the group of ten being summed). Make sure to include the value of A in your answer. You don't have to prove uniqueness.