Multiple Choice: Indicate your answer in the box to the right of each question.

1. Evaluate $-3x^3 - (20 - x^4)$ when $x = -2$
(a) 20 (b) -10 (c) 28 (d) -12 (e) None of these

1.

2. Which day of the week comes 60 days after Tuesday?
(a) Wednesday (b) Thursday (c) Friday (d) Saturday (e) Sunday

2.

3. If $\frac{2}{5}x + \frac{9}{5}y = -3$ and $\frac{7}{5}x - \frac{1}{5}y = 2$, what is the value of $4x - 8y$?
(a) 20 (b) 24 (c) -16 (d) $-\frac{24}{5}$ (e) None of these

3.

4. What is the remainder when 2^{245} is divided by 11?
(a) 4 (b) 6 (c) 8 (d) 10 (e) None of these

4.

5. Troy writes down the number 1. Each minute, Troy writes down a number 1 more than twice the previously written number. Of all the numbers Troy writes, which is closest to 1000?
(a) 977 (b) 997 (c) 1017 (d) 1023 (e) None of these

5.

6. How many distinct positive factors does $2^4 3^3 4^2$ have?
(a) 21 (b) 27 (c) 45 (d) 60 (e) None of these

6.

7. If the roots of $y = x^2 - 5x + 2$ are s and t, compute the value of $\frac{s}{t} + \frac{t}{s}$.
(a) $-\frac{29}{10}$ (b) $\frac{20}{7}$ (c) $\frac{21}{2}$ (d) 7 (e) None of these

7.

8. If $(x + a)^2 = 49$ and $(x + b)^2 = 9$, how many possible values can $a - b$ take?
(a) 1 (b) 2 (c) 4 (d) Infinitely many (e) None of these

8.

9. Compute the number of intersections of the graphs of $y = |x^2 - 1|$ and $y = x + 1$
(a) 0 (b) 1 (c) 2 (d) 3 (e) None of these

9.

Short Answer: Write your answer and show your work in the space below each question.
Clearly indicate your final answer by drawing a box around it.

10. Simplify the expression: $\frac{x^2-5x+6}{x^3+4x^2} \cdot \frac{2x}{x-3} \cdot \frac{x^2+6x+8}{x^2-4}$

11. If $x + y = 4$ and $x^2 + y^2 = 10$, compute the value of $x^3 + y^3$.

12. If A represents a single digit in the equation $7^{11} = 1A77326743$, compute A.

13. List all ordered pairs of positive integers (x, y) that are solutions of $x^2 - y^2 = 51$

14. Compute the length of the longest altitude of the 5-6-7 triangle.

15. What is the smallest n such that $n!$ ends in 15 zeros?

16. What is the length of the diagonal of an isosceles trapezoid with sides 15, 17, 15, and 21?

Long Answer: Write your solution in the space below each question. Make sure you include sufficient justification.

17. Prove that the equation $x^2 = 3 + 7y^2$ has no solutions in integers.

18. Let $f(n)$ denote how many positive integers with digits 1 and/or 2 have n be the sum of their digits.
For example, $f(4) = 5$ because there are 5 such integers (1111, 112, 121, 211, and 22)

a. What pattern is made by the values in the sequence $f(1)$, $f(2)$, $f(3)$, $f(4)$, $f(5)$, … ?

b. Prove that the pattern holds forever.