Multiple Choice: Indicate your answer in the box to the right of each question.

1. If $x^2 - 2x - 15 = 0$, find the value of $(x-1)^2$
(a) -3 (b) 4 (c) 5 (d) 16 (e) 25

1.

2. If $a = 0.5$, $b = \frac{1}{3}$, and $c = 24$, find the value of $a^4bc^2 - ca^2$
(a) -12 (b) -6 (c) 6 (d) 12 (e) 18

2.

3. Which number is largest: 2^{3^4}, 2^{4^3}, 3^{2^4}, 4^{2^3}, 4^{3^2}
(a) 2^{3^4} (b) 2^{4^3} (c) 3^{2^4} (d) 4^{2^3} (e) 4^{3^2}

3.

4. A shuttle bus goes back and forth on a straight route 17 blocks long. If the bus started on one end and travelled 200 total blocks before stopping for driver change, how far from the starting point is the bus located?
(a) 4 blocks (b) 6 blocks (c) 8 blocks (d) 11 blocks (e) 13 blocks

4.

5. What is the least positive value of x which satisfies $\big||x-5|-5\big| = 2$
(a) 1 (b) 2 (c) 7 (d) 8 (e) 12

5.

6. What is the smallest four-digit number in the arithmetic sequence 75, 122, 169, 216, …
(a) 1012 (b) 1013 (c) 1014 (d) 1015 (e) 1016

6.

7. The quadratic equations $y = ax^2 + 5x + 5$ and $y = ax^2 - 3x - 11$ have one root in common. What is it?
(a) a (b) -5 (c) -2 (d) 5 (e) 9

7.

8. How many rectangles with integer sides have perimeter 142 and an area that is a multiple of 10?
(a) 6 (b) 7 (c) 13 (d) 14 (e) 26

8.

9. The parabolas $y = x^2 + 6x - 2$ and $y = -x^2 + 4x + C$ have vertices a distance of 5 apart. Find C.
(a) -15 (b) -7 (c) 0 (d) 7 (e) 15

9.

Short Answer: Write your answer and show your work in the space below each question.
Clearly indicate your final answer by drawing a box around it.

10. Simplify the expression: $\left(\frac{2x}{6x^2-5x+1}\right)\left(\frac{2x^2+5x-3}{7x^2+21x}\right)$

11. Find integer a for which $\frac{20}{a} = 1.17647\ldots$

12. What is the area of the circle inscribed in a 5-12-13 triangle?

13. Blake has received test grades of 87, 82, and 95. What average does she need on the next two tests to have an average of 90 over the five test grades?

14. What is the smallest possible area of a circle with perpendicular chords of lengths 20 and 17?

15. How many of the integers from 1 to 2017 (inclusive) contain at least two 2's as digits?

16. What is the sum of all integer values of x for which $\frac{3x+16}{x-3}$ is an integer?

Long Answer: Write your solution in the space below each question. Make sure you include sufficient justification.

17. Fractions and addition rarely mix well, but let's take a look at what happens when they do. Pick two natural numbers m and n and define an *arithmetic fraction* $a(m,n)$ as follows:

$$a(m,n) = \frac{m + (m+1) + \cdots + ??}{1 + 2 + \cdots + n}$$

Where the number of consecutive terms being added in each sum is n.

a. What expression (in terms of m and n) should go in place of the "??" to ensure the top sum has n terms?
b. Find $a(7,5)$
c. State a closed formula for $a(m,n)$ that does not use ellipses ("…"). Simplify as much as you deem appropriate.
d. The smallest n for which $a(2017,n)$ is not an integer is $n = 4$. What is the second smallest value of n with this property?

18. The Stern-Brocot sequence can be formed as follows: Let $s_1 = \{0,1\}$. We keep forming the next sequence by inserting between each neighbors the value of their sum. So $s_2 = \{0,1,1\}$, $s_3 = \{0,1,1,2,1\}$, etc.

a. Find s_5
b. State the formula for the number of terms in s_n
c. The sum of the terms in s_n is $\frac{3^{n-1}+1}{2}$. Prove this result.