Multiple Choice: Indicate your answer in the box to the right of each question.

1. Evaluate: $100/4 - (7 - (1 - 6))$

(a) -8 (b) 12 (c) 13 (d) 23 (e) 27

1.

2. If $a = 4$, $b = \frac{1}{2}$, $c = 2$, $d = 3$, find the value of $ad - bc$

(a) -4 (b) 6.5 (c) 9.5 (d) 11 (e) 13

2.

3. What is the arithmetic mean of $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{7}{4}$?

(a) $\frac{35}{32}$ (b) $\frac{7}{8}$ (c) $\frac{17}{8}$ (d) $\frac{21}{16}$ (e) $\frac{21}{8}$

3.

4. A shuttle bus goes back and forth on a straight route 17 blocks long. If the bus started on one end and travelled 200 total blocks before stopping for driver change, how far from the starting point is the bus located?

(a) 4 blocks (b) 6 blocks (c) 8 blocks (d) 11 blocks (e) 13 blocks

4.

5. Express in simplest form: $\sqrt{(72)(490)}$

(a) $42\sqrt{20}$ (b) $84\sqrt{5}$ (c) $12\sqrt{245}$ (d) $420\sqrt{2}$ (e) 35,280

5.

6. How many different pizzas can have zero, one or two of seven available toppings? (The order of the toppings does not matter.)

(a) 21 (b) 22 (c) 28 (d) 29 (e) 128

6.

7. What is the smallest four-digit number in the arithmetic sequence 75, 122, 169, 216, …

(a) 1012 (b) 1013 (c) 1014 (d) 1015 (e) 1016

7.

8. How many rectangles with integer sides have perimeter 142 and an area that is a multiple of 10?

(a) 6 (b) 7 (c) 13 (d) 14 (e) 26

8.

9. 2017 has different digits that sum to 10. How many five-digit numbers have different digits that sum to 10?

(a) 72 (b) 84 (c) 96 (d) 126 (e) 252

9.

Short Answer: Write your answer and show your work in the space below each question.
Clearly indicate your final answer by drawing a box around it.

10. Solve for x: $2(x+7)+5(x-1)=(x+2)(x+3)-(x-2)(x-3)$

11. Factor completely: $3x^5+24x^4+48x^3$

12. Simplify the expression: $\left(\frac{2x}{6x^2-5x+1}\right)\left(\frac{2x^2+5x-3}{7x^2+21x}\right)$

13. Ren is giving presents to his neighbors. He has four gift boxes in colors red, white, black and yellow and four ribbons in the same colors (he only has one of each box and ribbon). In how many ways can he put the ribbons on the boxes so that each box gets one ribbon and no box gets a ribbon of the same color as itself?

14. Solve for a, b, c, d, and e: $\begin{cases} a+b=13 \\ b+c=6 \\ c+d=5 \\ d+e=6 \\ e+3a=14 \end{cases}$

15. Oscar's pasture is surrounded by a 2017 yard fence on the perimeter. Oscar wants to put up additional fence to subdivide his pasture into 25 identical smaller pastures in a five by five grid. How much additional fence does Oscar need?

16. The product of 2017 consecutive integers is 0. What is the greatest possible value of the arithmetic mean of those integers?

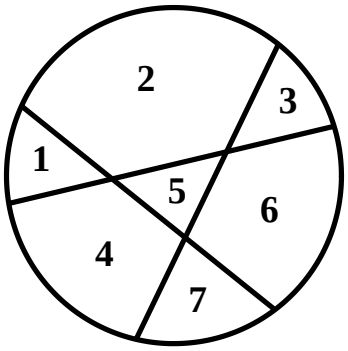
17. What is the largest integral factor of 111,111,111 which is less than 111,111,111? (The commas are for readability only and are not part of the number.)

18. The 5-12-13 right triangle has a circumscribed circle (outside circle through the vertices) and an inscribed circle (inside circle tangent to the sides). What is the ratio of the radii of the bigger circle to the smaller circle?

Long Answer: Write your solution in the space below each question. Make sure you include sufficient justification.

19. When chords are drawn in a circle, they divide the area into a number of regions. Let $f(n)$ be the maximum number of regions formed by n chords (for $n \geq 1$). For example, $f(3) = 7$ as shown on the right.

a. Find $f(5)$
b. Find the value of k such that $f(k) = 2017$
c. State a closed formula for $f(n)$ in terms of n.



20. Fractions and addition rarely mix well, but let's take a look at what happens when they do. Pick two natural numbers m and n and define an *arithmetic fraction* $a(m,n)$ as follows:

$$a(m,n) = \frac{m + (m+1) + \cdots + ??}{1 + 2 + \cdots + n}$$

Where the number of consecutive terms being added in each sum is n.

a. What expression (in terms of m and n) should go in place of the "??" to ensure the top sum has n terms?
b. Find $a(7,5)$
c. State a closed formula for $a(m,n)$ that does not use ellipses ("…"). Simplify as much as you deem appropriate.
d. The smallest n for which $a(2017,n)$ is not an integer is $n = 4$. What is the second smallest value of n with this property?