

- Levels Green, Blue and Black are graded separately.
- The exam consists of short questions and long questions.
- You may answer questions in multiple levels - we recommend you choose two, but please complete all questions in at least one level to enhance your chances of being admitted.
- Regardless of the number of levels you attempt, you must complete this exam within the two-hour window. There will be an additional 12-minute grace period for printing, scanning, and emailing the test to info+exam@nymathcircle.org. If you don't have a scanner, you may email a picture to us.
- A timer is started once you complete the Google Form - you will receive occasional e-mail reminders on time remaining. The time stamp on your submitted exam is proof of time taken on the exam.
- You will be admitted to the highest level you qualify for.
- **Sign the statement at the end of this document.**

use for scratch:

use for scratch:

Short Answer: Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a box around it.

1. Simplify the expression $2x - 3(x - 2) + (x - 2(2x - 1))$.

2. What is the area of an isosceles trapezoid with sides 5, 4, 5, and 10?

3. If $(x + a)^2 = 49$ and $(x + b)^2 = 9$, write down **all** possible values of $a - b$.

4. Patricia bicycled from City A to City B at an average rate of 12 miles per hour (mph) and returned over exactly the same route from City B to City A at an average rate of 16 mph. The entire trip took 7 hours. Find the distance from City A to City B.
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5. A convoy of trucks carrying 2800 lbs. each started to cross a desert. When two trucks broke down and had to be abandoned their cargo was distributed equally between the other trucks, resulting in each working truck carrying 224 lbs. more. How many trucks were in the original convoy?
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6. Compute the number of teams consisting of 3 boys and 2 girls that can be formed if there are 4 boys and 5 girls from which to select, and Robert (one of the boys) must be on the team.
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Long Answer: Write your solution in the space below each question.

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- (a) Compute 25^2 , 35^2 , 45^2 , 55^2 , and 65^2 . Can you use this pattern to quickly calculate 75^2 , 105^2 , 255^2 ?
- (b) Use the fact that $45^2 = 2025$ to find the prime factorization of 2021.
- (c) Is 100100009 prime? Justify your answer (Hint: You may find parts (a) and (b) useful for this problem).

use for scratch:

use for scratch:

Short Answer: Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a box around it.

1. If $x^2 - 5x + 2 = 0$ has roots s and t , compute the value of $\frac{s}{t} + \frac{t}{s}$.

2. Compute the number of intersections of the graphs of $y = |x^2 - 1|$ and $y = x + 1$.

3. A triangle has an area of 5 and a perimeter of 10. If the three altitudes have lengths h_1 , h_2 and h_3 , what is $\frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3}$?

4. List all ordered pairs of positive integers (x, y) that are solutions of $x^2 - y^2 = 51$.

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5. What is the remainder when 2^{2021} is divided by 11?

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6. Compute the highest power of 9 that divides $2021!$.
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Long Answer: Write your solution in the space below each question. Make sure you include sufficient justification.

7. Let $f(n)$ denote how many positive integers with digits 1 and/or 2 have n be the sum of their digits. For example, $f(4) = 5$ because there are 5 such integers (1111, 112, 121, 211, and 22)
- (a) What pattern is made by the values in the sequence $f(1), f(2), f(3), f(4), \dots$?
- (b) Prove that the pattern holds forever.

Use for scratch:

Use for scratch:

Short Answer: Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a box around it.

1. $2 \cos x, 3 \sin x, 4 \tan x$ form a geometric sequence (in that order) with $\sin x \neq 0$. Find $\sin x$

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2. In a circle of radius r a chord of length 24 splits the perpendicular diameter into segments whose lengths are in a ratio of 1:9. Compute r .

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3. How many of the integers from 1 to 2021, inclusive, can be represented as the difference of two squares?.

4. What is the smallest positive power of x that does NOT appear in the expansion of $(1 + x + x^3)^{10}$.
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5. $\tan A = \frac{\cos 2021^\circ + \sin 2021^\circ}{\cos 2021^\circ - \sin 2021^\circ}$ and $0^\circ \leq A < 180^\circ$. Compute A .
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6. Compute the remainder when $x^{2021} + 1$ is divided by $x^2 - 1$.
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Long Answer: Write your solution in the space below each question. Make sure you include sufficient justification.

7. The Fibonacci numbers are defined as $F_1 = F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. Show that there exists an integer A such that the sum of any ten consecutive Fibonacci numbers can be represented as A times some Fibonacci number (possibly from the group of ten being summed). Make sure to include the value of A in your answer. You don't have to prove uniqueness.

I worked independently on this exam.

Name: _____