2018 Summer High School Program   Placement Exam for Level C   NAME:__________________

Multiple Choice: Indicate your answer in the box to the right of each question.

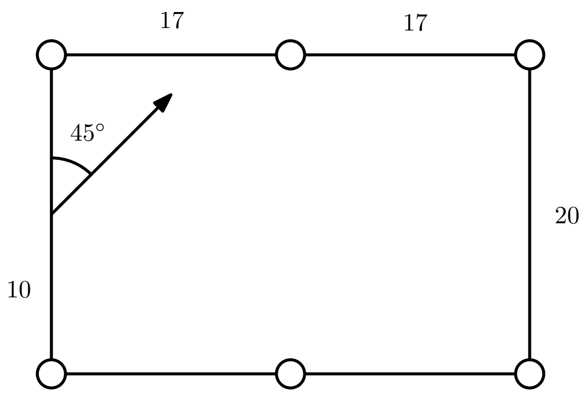
1. If $-1 \le a < 2$ and $-2 \le b \le 3$, find the minimum value of ab. (A) -6 (B) -5 (C) -4 (D) -3 (E) Does not exist	
2. Find the value of $(x+1)^2(x+5)+(x+3)^2(x+5)+(x+1)^2(x+3)^2$ when $x=-3$. (A) -8 (B) -7 (C) 7 (D) 8 (E) 16	
3. Find the last three digits of $35,347 \cdot 32,952 \cdot 62,315$. (A) 305 (B) 344 (C) 360 (D) 420 (E) 880	
4. Points A, B, C, D are in a plane so that $AB = BD = AD = CD = 20$ and $BC = 20\sqrt{2}$. Find $m\angle BCA$. (A) $22.5°$ (B) $30°$ (C) $45°$ (D) $60°$ (E) $75°$	
5. Find the sum $\cos^2 5° + \cos^2 10° + \cos^2 15° + \ldots + \cos^2 85°$. (A) $\frac{4\sqrt{3}}{2}$ (B) $4\sqrt{2}$ (C) $\frac{17}{2}$ (D) $\frac{17\sqrt{3}}{2}$ (E) $7\sqrt{2}$	
6. How many factors of 30! are perfect cubes? (A) 270 (B) 300 (C) 405 (D) 540 (E) None of these	
7. If $\sin\alpha, \sin 2\alpha, \sin^2\alpha$ is a geometric sequence and $0° < \alpha < 90°$, find $\sin\alpha$. (A) $\frac{\sqrt{17}-1}{4}$ (B) $\frac{\sqrt{17}-1}{8}$ (C) $\frac{\sqrt{65}+1}{4}$ (D) $\frac{\sqrt{65}-1}{4}$ (E) $\frac{\sqrt{65}-1}{8}$	
8. $\overline{AB}$ is a diameter of circle O and chord $\overline{PQ}$ intersects $\overline{AB}$ at X and is perpendicular to it. If $AO = 3PX$, find $\frac{OX}{XB}$. (A) $\frac{3\sqrt{2}-4}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{4}$ (D) $\frac{\sqrt{3}}{3}$ (E) $8+6\sqrt{2}$	

Short Answer: Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a $\boxed{BOX}$ around it.

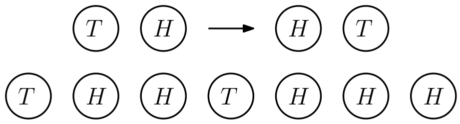
1. Three standard fair six-sided dice are rolled simultaneously and the numbers on their top faces are multiplied. What is the probability that the resulting number is not divisible by 6?

2. Let $S = \{3, 7, 11, 15, 19, 23, 27\}$. For how many 3-element subsets of S is the sum of their elements divisible by 3?

3. The pool table below has the specified dimensions. A ball is shot at a 45° angle as in the diagram. Which pocket will the ball land in?



4. In the diagram below, you are allowed to only flip over pairs of adjacent coins. I.e., you may flip only two coins at a time (see diagram). Is it possible to make all the coins Tails? Prove your claim.



5. When $(a+b+c)^{99}+(a-b+c)^{99}$ is fully expanded and all the like terms are combined, how many terms are there?

6. In ΔABC the inscribed circle is tangent to $\overline{AB}, \overline{BC}, \overline{AC}$ at P, Q, R, respectively. If $AB = 13, BC = 14, AC = 15$, find PR.

Long Answer: Write your solution in the space below each question. Make sure you include sufficient justification.

1. Find, with proof, all positive integers n such that $3^n - 104$ is a perfect square.

2. Given two sets A and B, a *bijective correspondence* between them is a rule that pairs every element of A with exactly one element of B and every element of B with exactly one element of A.

   (a) Let $A = \{1, 2, 3, 4\}, B = \{1, 4, 9, 16\}$. A bijective correspondence between A and B is $x \leftrightarrow x^2$. Another bijective correspondence (written explicitly) is $\{(1, 4), (2, 9), (3, 16), (4, 1)\}$. Give an example of another bijective correspondence between A and B.

   (b) Let $X = [1, 2], Y = [1, 4]$. A bijective correspondence between X and Y is $x \leftrightarrow x^2$. Give an example of another bijective correspondence between X and Y.

   (c) Give an example of a bijective correspondence between $X = \{1, 2, 3, \ldots\}$ and $Y = \{7, 8, 9, 10, \ldots\}$.

   (d) Give an example of a bijective correspondence between $X = (0, 1)$ and $Y = (1, \infty)$.

   (e) Give an example of a bijective correspondence between sets $X = [1, 2]$ and $Y = (1, 2]$.