2018 Summer High School Program Placement Exam for Level A NAME:_____

Multiple Choice: Indicate your answer in the box to the right of each question.

1. A certain person has figured out that he can tile the square shaped floor in his bathroom with square tiles. If 64 tiles are needed to tile the border between the floor and the walls, how many tiles are needed to cover the whole floor, including the border? (A) 128 (B) 225 (C) 289 (D) 900 (E) 1024	
2. Which of the following fractions is closest to 1? (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{5}{4}$ (D) $\frac{6}{7}$ (E) $\frac{11}{13}$	
3. If $2^{2x} = 4$ and $(-2)^{5y} = -32$, then what is the value of $(-2)^{x-y}$? (A) -4 (B) -1 (C) $-\frac{1}{4}$ (D) $\frac{1}{4}$ (E) 1	
4. If December 3rd of a certain year was a Sunday, what day of the week is January 25th of the next year? (A) Monday (B) Tuesday (C) Wednesday (D) Thursday (E) Friday	
5. What is the degree measure of the angle between the hour and minute hands on an analog clock at 7:30? (A) 15 (B) 30 (C) 40 (D) 45 (E) 60	
6. Determine the number of distinct positive factors of 1001. (A) 2 (B) 4 (C) 6 (D) 7 (E) 8	
7. A fair coin is tossed 5 times. What is the probability that it will land tails at least once? (A) $\frac{1}{32}$ (B) $\frac{5}{32}$ (C) $\frac{13}{16}$ (D) $\frac{27}{32}$ (E) $\frac{31}{32}$	
8. The degree measure of an interior angle of a regular polygon is a perfect square. How many sides does the polygon have? (A) 8 (B) 10 (C) 12 (D) 18 (E) 24	

Short Answer: Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a $\boxed{BOX}$ around it.

1. Find all values of x for which the below expression is undefined:

$$\frac{x^2+4}{\sqrt{x^2-4}}$$

2. Find the sum

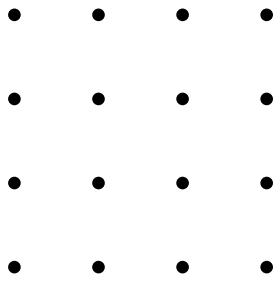
$$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{2}{3}+\frac{2}{4}+\frac{2}{5}+\frac{2}{6}+\frac{2}{7}+\frac{3}{4}+\frac{3}{5}+\frac{3}{6}+\frac{3}{7}+\frac{4}{5}+\frac{4}{6}+\frac{4}{7}+\frac{5}{6}+\frac{5}{7}+\frac{6}{7}$$

3. What is the last digit of 3^{2018}?

4. In the middle of the Sahara desert there is a tribe of Hungry Sand Eaters. Two Hungry Sand Eaters can eat 10 pounds of sand in 4 days. How much sand will 5 Hungry Sand Eaters eat in 6 days?

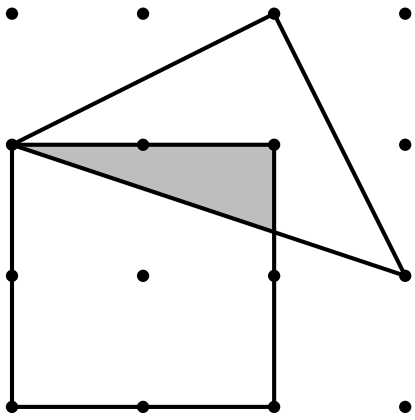
5. Factor completely: $3x^5 - 6x^3y^2 + 3xy^4$

6. How many rectangles can be formed with the vertices on the below lattice? [Note: The sides of the rectangles do not necessarily have to be horizontal or vertical.]



7. If $x + y = 2$ and $x^2 + y^2 = 3$, find the value of $x^3 + y^3$.

8. Find the area of the shaded region:



Long Answer: Write your solution in the space below each question. Make sure you include sufficient justification.

1. Let $f(n)$ be the maximum number of intersections that $n \geq 2$ circles can have. For example, $f(2) = 2, f(3) = 6$, and so on.

   (a) Find $f(5)$.

   (b) Find an explicit formula for $f(n)$.

2. (a) Show that $(x+1)|(x^4-1)$ (i.e., $(x+1)$ is a factor of x^4-1)

   (b) Show that $(x^2+x+1)|(x^6-1)$

   (c) Show that if $(m+1)|n$, then

$$(1+x+x^2+\ldots+x^m)|(x^n-1)$$