Fall

- Paper folding: equilateral triangle (parallel postulate and proofs of theorems that result, similar triangles), Trisect a square paper

- Divisibility by 2-11 and by combinations of relatively prime numbers such as 3 and 4 for 12, etc. including word problems related to the rules

- Introduction to Cartesian coordinate plane

- Sets: unions, intersections, complements, Venn Diagrams, Set builder notation

- Counting: factorials, permutations, combinations, Pascals Triangles, path problems

Spring

- Operations in Binary

- Operations in bases other than ten

- Pattern: finding the units digit, perfect numbers, sequences

- Series: sum of consecutive integers and consecutive cubes (geometrically)

- Proofs of the Pythagorean Theorem, tilted squares

- Area: Right and Isosceles Triangles, Trapezoids, Regular Polygons, Circles

- Factors, Multiples, Divisibility Rules

- Modular Congruence and Properties (Proofs of divisibility rules and proofs of properties)

Possible Summer Topics

- More modular congruence

- Cryptography Cease Cipher, Keyword Cipher, Letter to Number cipher, Pigpen Cipher, word Shift Cipher, Ciphers and Modulus, Public Key Cryptography

- Sequences

- Problems from UKMT math challenges

Sample Problems

1. Which of the five diagrams below could be drawn without taking the pen off the page and without drawing along a line already drawn? Can you explain your reasoning? (nrich maths)
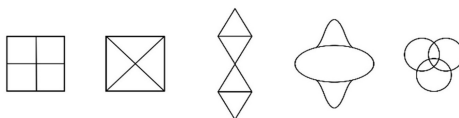
2. In how many ways can we form a license plate using only digits (zero through nine) and capital letters other than O and I, given that each plate has six characters, the first of which is a digit, and the second of which is a letter?

3. How many four digit positive integers are there such that the leftmost digit is odd, the second digit is even, and all four digits are different? (AMC 8)

4. Choose any three digits and make a six digit number by repeating the three digits in the same order e.g. 123123. Is your number divisible by 7? 11? 13? Why?

5. Find the number of paths from point A to point B if you are only allowed to travel east or south.
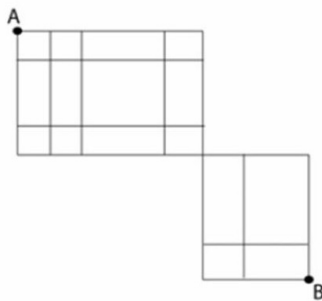
6. How big is angle x?
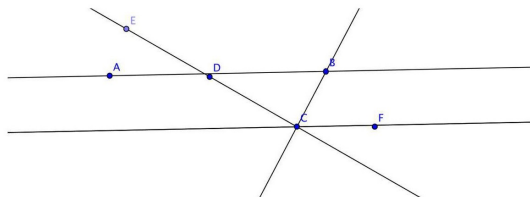
$$m\angle ADE = 36^{o}; \quad m\angle BCF = 80^{o}; \quad m\angle DCB = x^{o}; \quad \overrightarrow{AD}//\overrightarrow{CF}$$

7. Calculate the binary numbers. Describe the pattern.

   (a) 11+11

   (b) 111+111

   (c) 1111+1111

   (d) 11111+11111

8. Calculate the binary numbers. Describe the pattern.

   (a) 10+10

   (b) 100+100

   (c) 1000+1000

   (d) 10000+10000

9. Convert the following to base 10:

   (a) 412_5

   (b) 1471_8

   (c) 1210_3

   (d) 612_7

10. Convert the following numbers from base 10 to the base stated

    (a) 24 to base 3

    (b) 321 to base 5

    (c) 314 to base 7

    (d) 16 to base 4

    (e) 84 to base 9

11. In which base was each of the following calculations carried out?

    (a) $4_b + 2_b = 11_b$

    (b) $7_b + 5_b = 13_b$

    (c) $8_b + 2_b = 17_b$

    (d) $4_b \times 5_b = 32_b$

    (e) $11_b - 3_b = 5_b$

    (f) $22_b - 4_b = 13_b$

12. Change 147 in base 8 into a base 3 number.

13. Change 321 in base 4 into a base 7 number.

14. (a) Ten mathematicians met up one week. The first mathematician shook hands with all the others. The second one shook hands with all the others apart from the first one (since they had already shaken hands). The third one shook hands with all the others apart from the first and the second mathematicians, and so on, until everyone had shaken hands with everyone else. How many handshakes were there altogether?

    (b) One Hundred mathematicians met up one week. The first mathematician shook hands with all the others. The second one shook hands with all the others apart from the first one (since they had already shaken hands). The third one shook hands with all the others apart from the first and the second mathematicians, and so on, until everyone had shaken hands with everyone else. How many handshakes were there altogether?

15. An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 4, what is the area of the hexagon?

16. A square with area 4 is inscribed in a square with area 5, with one vertex of the smaller square on each side of the larger square. A vertex of the smaller square divides a side of the larger square into two segments, one of length ar , and the other of length b . What is the value of ab ?
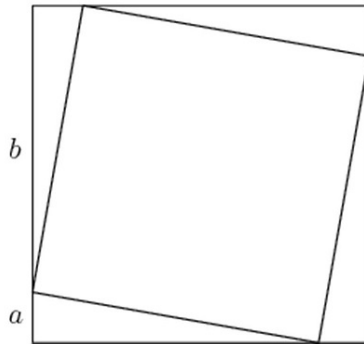
17. Consider mod p where p is prime. For each number that is not 0 mod$p$, there exists a multiplicative inverse. For each a not congruent to 0 mod p, there exists a,b such that $ab \equiv 1$ mod p.

    (a) Find the inverses of 1,2,3,4,5 and 6 mod 7

    (b) Find all the inverses of the above in mod 11 and mod 13

18. Which numbers have inverses in mod 4? Mod 6? Mod 8? Mod 10? Mod 16? Do you see any patterns?

19. Last year, George's age was a cube number. In a year's time, it will be a prime number. How old is George now?

20. Let a and b be any real numbers. We say that a is a multiple of b if $a =...$

21. Are 133 and 175 multiples of 7?

22. Is $133 + 175$ a multiple of 7?

23. Suppose that a and b are multiples of 7. Must a + b be a multiple of 7?

24. Suppose that q is a multiple of 7. Is q + 25 a multiple of 7?

25. Is every multiple of 10 also a multiple of 5?

26. Determine which of the following are multiples of 5:

    (a) 117_9

    (b) 111101_2

    (c) 4105_6

    (d) $A1BA_{15}$

27. Determine which of the following are multiples of 2:

    (a) 116_9

    (b) 111101_2

    (c) 4105_6

    (d) $A1BA_{15}$

28. How many primes are there between 1 and 50? List them.

29. (a) Write the prime factorization of 504

    (b) Find the smallest positive integer y such that $504y$ is a perfect square.

30. (a) If 3 is a factor of N and 4 is a factor of N, must 12 be a factor of N?

    (b) If 6 is a factor of N and 4 is a factor of N, must 24 be a factor of N?

31. Are 40 and 25 multiples of 5?

32. Is 40+25 a multiple of 5?

33. If you add two multiples of 5, do you get a multiple of 5?

34. If you add two multiples of 3, do you get a multiple of 3?

35. If a and b are multiples of c then $a + b$ and $a - b$ are multiples of...

36. 30 is a multiple of 3. If $30 + x$ is also a multiple of 3, what must be true about x?

37. More generally, if $a \mid b$ and $a \mid (b + c)$, must it be true that $a \mid c$?

38. If 3 is a factor of $(a + b)$, must 3 be a factor of both a and b?

39. If $2^x \cdot 3^y \cdot 5^z = 54,000$, find $x + y + z$