Year 1: Fall

- Pigeonhole 1
- Pigeonhole 2
- Induction 1
- Induction 2
- Inequalities 1 (AM-GM)
- Geometry 1 - Triangle Area Ratio Theorem (TART)
- Contest (Math Battle)
- Geometry 2 - Inscribed Quadrilaterals, Ptolemy's Theorem, Brahmagupta's Theorem
- Mistakes in Geometry proofs

Year 1: Spring

- Number Theory
- Contest (Math Battlle)
- Basic Counting
- One to One Counting
- Generating Functions 1
- Generating Functions 2
- Expected Value 1
- Expected Value 2
- Principle of Inclusion/Exclusion (PIE)
- Geometric Transformations

Year 2: Fall

- Number Theory 1 - Fermat's Little Theorem, Euler's Generalization
- Number Theorey 2 - Chinese Remainder Theorem
- Polynomials 1 - Simon's Favorite Factoring Trick, Vieta's Formulas
- Polynomials 2 - Roots of Unity and Euler's Formula
- Recursion 1
- Contest (Math Battle)
- Wrong proofs in Number Theory

Year 2: Spring

- Events with States
- Graph Theory 1
- Graph Theory 2
- Number Theory - Dvisibility
- Number Theory - Euclid's Algorithm
- Number Theory - Modular Arithmentic
- Random Walks
- Contest (Math Battle)

Possible Summer Topics

- Methods of Proof
- Contradiction
- Extreme Principle
- Invariant
- Pigeonhole Principle
- Introduction to Graphs
- Complex Numbers 1
- Vectors
- Trigonometry - triple angle formulas, identities, substitution in algebraic problems.
- Geometric Probability
- Diophantine Equations - Decomposition and Inequalities
- Diophantine Equations - Modular Arithmetic and Infinite Descent
- Game Theory - 2 player games, Single Pile games, Nim and other advanced games.

Sample Problems

1. In a unit square there are 5 points. Prove that some two points are at most $\frac{\sqrt{2}}{2}$ units apart.
2. A chessmaster has 77 days to prepare for a tournament. She wants to play at least 1 game every day, but no more than 132 games in total. Prove that there is a sequence of successive days in which she plays precisely 21 games in total.
3. Into how many regions do n lines in general position divide a plane? Lines are in general position if no two of them are parallel and no three intersect at a point.
4. (IMO 1997) An $n \times n$ array whose entries come from the set $S = \{1, 2, ..., 2n - 1\}$ is called a *silver* matrix if for each $i = 1, 2, ..., n$, the ith row and ith column together contain all the members of S. Show that silver matrices exist for infinitely many n. Here is an example of silver matrix:
$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$
5. Prove that for non-negative x and y, $x^3y^5 \leq \frac{3}{8}x^8 + \frac{5}{8}y^8$. Use this result to show that
$$x^8 + y^8 \geq x^3y^5 + x^5y^3$$
6. Using TART, prove
   - Ceva's Theorem
   - Angle Bisector Theorem
   - Stweart's Theorem
   - Length of median formula
   - The area of the triangle formed by the medians of triangle T is $\frac{3}{4}$ the are of triangle T.
7. Prove that in any convex quadrilateral, the segments connecting the midpoints of the opposite sides bisect each other. This point of intersection is called the *centroid* of the quadrilateral.
8. Prove or disprove: For any positive $k \geq 1, (6k + 5, 7k + 6) = 1$.
9. How many orderings of letters of MISSISSIPPI are palindromes?
10. What is the maximum number of intersection points of diagonals inside a convex n-gon?
11. In how many ways can we roll 3 dice to get thhe sum 9?
12. Show that the number of partitions of n that have no number repeated equals the number of partitons of n that have only odd numbers. Fore example, if n=4, partitions with no number repeated ar 4 and 3+1 and partitions with only odd numbers are 3+1 and 1+1+1+1.

13. I have n letters and n addressed envelopes. If I place letters into the envelopes at random, what is the expected number of correctly addressed envelopes?
14. (HMMT) Values $a_1, a_2, ..., a_{2013}$ are chosen independently and at random from the set $\{1, 2, ..., 2013\}$. What is the expected number of distinct values in the set $\{a_1, a_2, ..., a_{2013}\}$
15. Thirty students wish to enroll in one of three classes: chemistry, physics or biology. In how many ways can they enroll if no class can be empty?
16. Squares are erected on the sides of a parallelogram externally. Prove that the centers of the squares form a square.
17. Prove that there esist two irrational numbers a and b such that a^b is rational.
18. There are 1000 people playing laser tag. At a signal, each person shoots the person closest to them. It just so happens that at the signal, the pairwise distances between all the players are distinct, so there is no ambiguity. Prove that no person is shot by more than 5 players.
19. On every one of the 15 planets, there is a stargazer who observes the closest planet. The pairwise distances between planets are all distinct. Prove that there is some planet that isn't observed by anyone.
20. At the coat check there are 100 coats. Every person who comes to the coat check either picks up or leaves a coat. After 25 people came to the coat check, is it possible that there are 90 coats in the coat check?
21. Prove that any $(n+1)$-element subset of $2n$ consecutive natural numbers contains a pair of relatively prime integers.
22. In a newly constructed neighborhood, some houses are connected by direct phone lines. In an attempt to integrate the neighborhood, the Board of Community Improvement and Integration wants to sell houses in such a way that each white family is directly connected with at least as many African-American families as white families, and vice versa. Can the Board do this?
23. Determine the locus of points satisfying
$$Re\Big(\frac{z-1-i}{z+1+i}\Big) = 0$$
24. Find a closed formula for
$$\cos\theta + \cos 3\theta + ... + \cos(2n-1)\theta$$
25. Given a parallelogram $ABCD$, express $\vec{AB}$ in terms of $\vec{AC}$ and $\vec{BD}$
26. Points P,Q,R are chosen on sides AB,BC,CA of a triangle ABC, respectively, so that
$$AP : PB = BQ : QC = CR : RA$$
Prove that centroids of ABC and PQR coincide.

27. (IMO '63) Prove that if all the angles of a convex n-gon are equal and the lengths of consecutive edges $a_1, ..., a_n$ satisfy $a_1 \geq a_2 \geq ... \geq a_n$, then the polygon is regular.

28. Prove that $x^4 + y^4 + z^4 = 2009$ has no integer solutions.

29. Prove that $437|18! + 1$

30. (1987 AIME No.5) m, n are integers such that $m^2 + 3m^2n^2 = 30n^2 + 517$. Find $3m^2n^2$

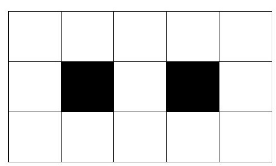
31. Factor $z^5 + z + 1$

32. (AIME 1996 No.9) I have 2048 letters numbered 1 to 2048. I have to address every single one, starting with No.1. When I address a letter, I put it in my outbox. The ones I skip I stack as I skip them (so No.2 is on the bottom of the stack after my first pass). After I finish my first pass, I have 1024 letters which are not addressed; No.2048 is on top, No.2 is on bottom. I then repeat my procedure over until there's only one letter left. What number is that letter?

33. In a best-of-7 series, two baseball teams, Team A and Team B, play against each other until one of the teams wins a total of 4 games. If the probability that Team A will beat Team B in any given game is 2/3, what is the expected number of games in the series.

34. Prove that a tree with n vertices has exactly n-1 edges.

35. A snail decided to crawl on a 3×5 board with two squares removed (see figure). Can it visit every square just once and then end up where it started?



36. If $n > 1$, then $n^4 + 4^n$ is never a prime.

37. Find integers x, y such that

$$754x + 221y = (654, 221)$$

38. What are the last two digits of 3^{1234}?

39. A bug lives on the vertices of a tetrahedron and each second jumps to one of the three adjacent vertices with equal probability. What is the probability that after 6 jumps the bug will be back at its original vertex?

40. Prove there are infinitely many primes

41. Derive and prove the triple angle formulas for $\sin 3\alpha, \cos 3\alpha, \tan 3\alpha$.

42. Prove that in $\triangle ABC$, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

43. Prove the inequality $\sqrt{ab} + \sqrt{(1-a)(1-b)} \leq 1$ for $a, b \in [0, 1]$

44. Two points are chosen uniformly at random on $[0, 1]$. What is the expected distance between them?

45. Solve in positive integers:

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{17}$$

46. (Balkan MO) Prove that $x^5 - y^2 = 4$ has no solutions in integers.