Year 1: Fall

- Divisibility properties and problems
- Factorization of Integers, counting the number of factors of an integer
- The Euclidean Algorithm and Linear Diophantine Equations
- GCD/LCM Problems for two or more numbers. Derive general solutions to a linear Diophantine Equation.
- Arithmetic operations in other bases
- Combinatorics: multiplicative counting principle, complementary counting, casework.

Year 1: Spring

- Pythagorean Triangles and Triples.
- Greatest Integer Function and its applications.
- Modular Arithmetic and Fermat's Little Theorem
- Coordinate Geometry
- Combinatorics: Permutations and Combinations
- Binomial Theorem and Pascal's Triangle
- Sums of Powers of Natural Numbers

Year 2: Fall

- Probability and Combinatorics: symmetry principle in probability, multinomail coefficients.
- Partitions
- Multinomial Theorem
- Factorization of Polynomials: difference of squares, Sophie Germain's identity, sum and difference of cubes
- Rational Root Theorem (RRT)

Year 2: Spring

- Triangle Trigonometry
- Remainder Theorem
- Recursive functions
- Absolute Value Equations
- Circles: power of a point
- Sequences and Series: arithmetic and geometric sequences and series

Sample Problems

1. Derive a divisibility property for 13
2. Find all ordered pairs (A, B) such that $A23774B$ is divisible by 36.
3. Find the number of positive divisors of 3240.
4. Find the prime factorization of 999991
5. Given a 1000×1000 multiplication table, how many times does the number 288 appear?
6. Find the greatest common divisor of 3234 and 5264 using the Euclidean algorithm.
7. Solve $5x + 13y = 19$ over the integers.
8. Explain how 8 gallons of water can be obtained using two containers with volumes 32 and 52 gallons.
9. Find all pairs of numbers a and b if $gcd(a, b) = 1989$ and $lcm(a, b) = 13$.
10. Find the general solution to the Diophantine Equation $19x + 13y = 23$.
11. If the product $(2^{51} + 1)(2^{50} - 1)$ is expressed in base 2, compute the number of 0's in the result. (NYSML)
12. Evalute the expression in base 5: $(134_5 + 201_5) \cdot 23_5$
13. How many postive integers less than 2018 can be made using only prime digits?
14. How many ways are there to position eight distinct rooks on a regular 8×8 chessboard such that no two rooks attack each other?
15. If $(1+3+5+\cdots+p)+(1+3+5+\cdots+q) = (1+3+5+\cdots+r)$, where each set of parentheses contains the sum of consecutive odd integers, compute the smallest possible value for the sum $p + q + r$, if $p > 7$. (NYSML)
16. The sides of a right triangle are all integers. Two of these integers are primes that differ by 50. Compute the smallest possible value for the third side. (ARML)
17. How many ending zeros does the number 2018! have?
18. Solve for x: $\left\lfloor 8x - \frac{1}{2} \right\rfloor = 9$.
19. Evaluate $\left\lfloor\sqrt{1}\right\rfloor + \left\lfloor\sqrt{2}\right\rfloor + \cdots + \left\lfloor\sqrt{2016}\right\rfloor + \left\lfloor\sqrt{2017}\right\rfloor$
20. Prove that $1^{2017} + 2^{2017} + \cdots + 15^{2017} + 16^{2017}$ is divisible by 17.
21. Find the remainder when 330^{9000} is divided by 311.
22. Prove that the equation $x^2 + y^2 = 59^{4n+59}$ has no solution in integers.

23. The vertices of a pentagon $ABCDE$ are given by the points $A(1,7), B(4,12), E(7,2), D(8,11)$, and $C(6,10)$. Find the area of the pentagon.

24. Two boys and four girls need to be picked from a group of six boys and severn girls to participate in a school play. How many different selections can be made?

25. If $\binom{1991}{991} + \binom{1991}{992} = 1992x$ and $x > 996$, compute x. (NYSML 1991)

26. Find the constant coefficient in the expansion of

$$\left(x^3 + \frac{2}{x}\right)^{20}$$

27. Given a three-digit number $4A1$ raised into the power of $1A4$. If the tens digit of $(4A1)^{1A4}$ is 2, compute all possible values for the digit A. (ARML)

28. Compute$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 99 \cdot 100$.

29. Three fair six-sided dice are rolled. What is the probability that the values shown aon two of the dice sum to the value shown on the remaining die? (AMC 10 2014)

30. Carl and Claire each roll a fair six-sided die. What is the probability that Claire rolls a higher number than Carl?

31. There are six investment options and up to \$20K to invest. Each investment must be in units of a thousand dollars. How many different ways to invest the money are possible if it is not required to use each investment option?

32. Solve the equation $a + b + c = 24$ in positive integers.

33. How many terms does a completely simplified expansion of $(a + b + c)^{11}$ contain?

34. What is the coefficient of the term containing ab^2c^3 in the expansion of $(a + 2b - 3c)^6$?

35. Factor $x^4 + 64y^8$ over integers

36. Factor $x^6 - 9x^4 - x^3 + 27x^2 - 27$ over integers.

37. Solve $x^3 - 79x + 210 = 0$

38. Prove that the sum of the squares of the distances from the vertex of the right angle, in a right triangle, to the trisection points along the hypotenuse is equal to $\frac{5}{9}$ the square of the measure of the hypotenuse. (Challenging Problems in Geometry 2)

39. In quadrilateral $ABCD$ no pair of opposite sides is parallel. The acute angle between the diagonals of the quadrilateral is 75°. Find the exact area of $ABCD$ if the lengths of the diagonals are 8 cm and 13 cm.

40. A polynomial $P(x)$ leaves the remainder of -11 when divided by $x - 4$ and the remainder of 16 when divided by $x + 5$. Find the remainder left after dividing $P(x)$ by $(x - 4)(x + 5)$.

41. Find the number of ways to tile a 10×1 strip using only 1×1 squares or 2×1 dominoes.

42. Evaluate
$$x = \sqrt{2017 + \sqrt{2017 + \sqrt{2017 + \cdots}}}$$

43. Solve $|2x+3| - |3x-5| = 4$

44. A circle has two parallel chords of length x that are x units apart. If the part of the circle included between the chords has area $2+\pi$, find x. (HMMT)

45. In an arithmetic progression, the ratio of the sum of the first r terms to the sum of the first s terms is equal to the ratio of r^2 to s^2 $(r \neq s)$. Compute the ratio of the 8^{th} term to the 23^{rd} term. (ARML)