Curriculum for a two year cycle

We assign categories NT (number theory), A (algebra), C (combinatorics) and G (geometry) to the topics. This the most prominent topics used to classify and diversify problems in most of the mathematical contest including USAMO and IMO

Year 1: Fall.

(1) (NT+A) Reminder: Arithmetics modulo n,
   (a) Fermat's little theorem, Euler's theorem
   (b) Euler's totient function
   (c) Euclidean algorithm
   (d) RSA encryption algorithm*

(2) (C) Graphs
   (a) Trees & forests. Basic applications of Euler's formula $V - E = \#\{\text{components}\}$
   (b) Breadth-first and depth-first search*

(3) (G) Rigid motions of the plane and how to use them in geometric problems
   (a) Symmetries
   (b) Parallel transport
   (c) Rotations (central symmetries). Napoleon's theorem & Toricelli point

Year 1: Spring.

(1) (NT+A) Fermat's Christmas theorem
   (a) Gaussian integers
   (b) Fundamental theorem of arithmetics in $\mathbb{Z}[i]$
   (c) Number of representations of n as $a^2 + b^2$. Formula for $\pi/4$.
   (d) Zagier's proof 'from the book'
   (e) Quaternions & Hurwitz numbers. Lagrange's 4-squares theorem*

(2) (C) Catalan numbers
   (a) Various definitions & direct bijections.
   (b) Recurrence & generating function.
   (c) Explicit formula. Combinatorial and algebraic proofs.

(3) (C) Hall's marriage lemma
   (a) Proof and coloring of bipartite graphs
   (b) Hall's lemma in hypercube

(4) (G) Projective geometry
   (a) Projective line. Double relation
   (b) Projective plane. Double relation of lines & angles.
   (c) Desargues's theorem and Pappus's theorems. Projective duality.
   (d) (C) Finite projective planes*. Mathematics of 'Spot It!'

Year 2: Fall.

(1) (A) Polynomials
   (a) Review quadratic polynomials in and out.
   (b) Bezout theorem
   (c) Continuity & asymptotics at $\pm\infty$.
   (d) Vieta theorem
   (e) (NT) Vieta jumping and infinite descend*. Fermat's last theorem for $n = 4$.

(2) (C) Graphs
   (a) Ramsey theory
   (b) Planar graphs and Euler's formula
   (c) 5-colors theorem

(3) (A) Functional equations
   (a) Substitution. Importance of the range and domain
   (b) Cauchy equation over $\mathbb{Q}$ and $\mathbb{R}$

(4) (G+C) Extremal principle in geometry
   (a) Larges/smallest angle/side/distance/area
   (b) Convex hull & support lines

Year 2. Spring.

(1) (G) Homothety and rotational homothety
   (a) Homothetic circles
   (b) Composition of homotheties
   (c) Rotational homothety and its center

(2) (G+A) Pick's formula
   (a) Lattice polygons. Determinant
   (b) Functions on polygons and $SL_2(\mathbb{Z})$-invariance
   (c) Generalizations of Pick's formula & Ehrhart polynomial

(3) (C) Chromatic polynomial of a graph
   (a) Proof of polynomiality
   (b) Contraction-deletion relation
   (c) Circuits on graphs

(4) (C) Probabilistic methods
   (a) Notion of probability
   (b) Random variables and expectation
   (c) Linearity of expectation
   (d) Applications to combinatorial problems. Non-constructive proofs