# 2023/24 Placement Tests

Click on the level you want to test for:

<u>MSA</u> <u>MSB</u> <u>MSC</u> <u>HSA</u> <u>HSB</u> <u>HSC</u> <u>College Bridge</u>

## MSA PLACEMENT TEST(Arithmetic Proficiency)

Answer as many questions as you can. **Reduce all fractions to lowest terms**.

Name: \_\_\_\_\_\_Date: \_\_\_\_\_Date: \_\_\_\_\_

Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a box around it.

1. Calculate 265.3 - 187.7

2. What is 0.375 as a fraction (don't forget to reduce your answer to its lowest terms)

3. Calculate  $3\frac{3}{5} \times 3\frac{1}{3}$ 

<sup>4.</sup> If April has a \$20 bill and bananas cost \$0.65 each, how many bananas can she buy and how much will she have left over?

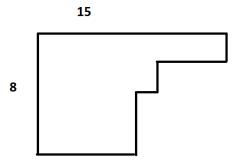
5. May gets 25 questions right on a test with 40 questions. What percentage did she get?

6. Jun buys a book for \$25. There is a sales tax of 6%. How much did she have to pay altogether?

7. Julia has \$12.40, Augustin has \$8.40 and Neve has \$1.40. If they shared their money so everyone has the same amount, how much did Neve receive?

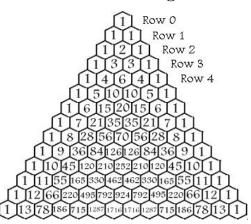
8. A row of fenceposts are evenly spaced around a square. The distance from one fencepost to the next is two feet. If the square is six feet by six feet, how many fenceposts are there?

9. Find the perimeter of this shape:



10. Matchias is catching fish. If he had caught three times as many fish as he has actually caught, he would have 12 more fish. How many fish has he caught?

#### Pascal's Triangle



## MSB PLACEMENT TEST

#### Answer as many questions as you can. Reduce all fractions to lowest terms.

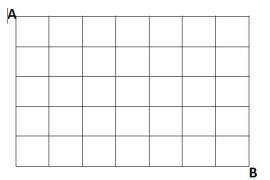
Name: \_\_\_\_\_\_Date: \_\_\_\_\_\_Date: \_\_\_\_\_\_

Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a box around it.

- 1. There are 20 people at the Smith family reunion. If each person hugs all the other people once, how many hugs will there be?
- 2. The 5-digit number 6836N is divisible by 9. What does the digit N represent?
- 3. Find the units digit of  $2^{2022}$
- 4. Ben spent one-fifth of his money buying a notebook for class. He spent another half of what was left for a haircut. Then, he bought lunch for \$10. When he got home, he had \$2 left. How much did he have originally?



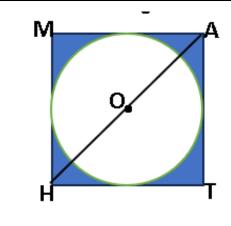
Find the number of pumpkins in (a) the 43rd image. (b) the nth image.



6.

How many different paths are there to go from point A to point B if you can only go down and to the right?

7. If each leg of a right triangle is 6 inches, find the exact length of the hypotenuse.

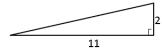


#### 8.

Given that circle O is inscribed in square MATH and diagonal HA measures  $\sqrt{18}$  , find the area of the shaded portion of the figure.

9. In how many ways can the letters of the word CIRCLE be arranged?

10. Find the exact perimeter of the right triangle below.



### MSC PLACEMENT TEST

Answer as many questions as you can. Reduce all fractions to lowest terms.

Name: \_\_\_\_\_Grade: \_\_\_\_\_Date: \_\_\_\_\_

Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a box around it.

1. What are the first 10 triangular numbers?

2. Using Fermat's Little Theorem, find  $128^{129} \mod 17$ . Show your work.

3. Convert  $130_6$  to base 10.

4. The sum of two numbers is 9 and their product is 12. What is the difference of the two numbers?

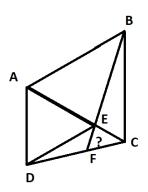
5. What is the smallest 5-digit integer divisible both by 8 and 9?

6. 452x8 is divisible by both 3 and 8. Find a possible value for x.

<sup>7.</sup> Of the 30 people on Anna's swim team, there are twice as many people who swim freestroke as backstroke. If there are 6 students who swim both freestroke and backstroke, and 3 students who swim neither freestroke nore backstroke, how many students swim backstroke?

8. How many different sets of 3 books can be chosen from a shelf of 20?

9.  $\triangle ABC$  and  $\triangle AED$  are equilateral. Calculate  $m \angle EFC$  (Hint: Extend DE to meet BC at G. What kind of quadrilateral is DABG? What can you say about triangles BGE and DFE?).



<sup>10.</sup> If you flip a coin 8 times, how many outcomes are possible? How does your answer relate to Pascal's Triangle?

#### HSA PLACEMENT TEST(Albegra Proficiency)

Answer as many questions as you can. Reduce all fractions to lowest terms.

Name: \_\_\_\_\_Grade: \_\_\_\_\_Date: \_\_\_\_\_

Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a box around it.

1. Solve (x+1)(x-1) = 2.

2. Simplify 
$$\frac{x-1}{1-\frac{1}{x}}$$
.

3. What is the domain of  $f(x) = \frac{1}{x-2}$ ?

4. Solve  $|x+3| \le 0$ 

5. Write  $y = 2x^2 - 6x + 1$  in vertex form.

6. Factor  $x^4 - 16$  completely.

7. Calculate  $(3\sqrt{3})^2 - (2\sqrt{2})^2$ , showing your work.

8. If 
$$f(x) = \frac{x-1}{x+3}$$
,  
(a) Calculate  $f(-1)$   
(b) Simplify  $f(x-1)$   
(c) Find  $f^{-1}(x)$ 

9. If John drives at 40 miles per hour to his friend's house and then 20 miles per hour to return, what was his overall average speed?

10. The price of a stock increased by 20% one year and then decreased by 20% the next. Was the net increase positive, negative or zero? Explain.

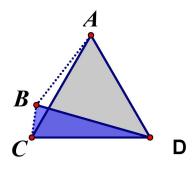
#### HSB PLACEMENT TEST

Answer as many questions as you can.	
Reduce all fractions to lowest terms.	

Name: \_\_\_\_\_\_Grade: \_\_\_\_\_\_Date: \_\_\_\_\_

Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a box around it.

1.  $\triangle ADC$  is equilateral and  $\triangle DBC$  is isosceles. Calculate  $\angle ABC$ :



- 2. Denote the floor of x by  $\lfloor x \rfloor$ . Compute:  $\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor \dots + \lfloor \sqrt{2020} \rfloor + \lfloor \sqrt{2021} \rfloor$
- 3. Determine the number of terminating zeroes in 8000!
- 4. Euclid's Formula is  $(m^2 n^2)^2 + (2mn)^2 = (m^2 + n^2)$  and is used for generating Pythagorean Triples. What values of m and n should be used to generate the following Pythagorean Triples?
  - (a) 16,30,34
  - (b) 10,24,26

5. There are two Pythagorean Triangles that have r, the radius of the inscribed circle, equal to 5. What are they? Which one has a bigger area?

6. Find the remainder when 27! is divided by 29 (Hint: use Wilson's Theorem).

7. In  $\triangle ABC$ , AB = 10, BC = 13, CA = 13, and  $\overline{\text{DC}}$  and  $\overline{\text{EA}}$  are the altitudes from C and A, respectively. Find DE.

8. Solve in the naturals:  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ 

9. If x and y are chosen randomly so that  $2 \le x \le 6$  and  $3 \le y \le 6$ , find the probability that  $x + y \le 9$ .

10. Use the Chinese Remainder Theorem to find the smallest positive solution to this system of congruences:

 $\begin{array}{ll} x \equiv 2 \mod 3 \\ x \equiv 3 \mod 5 \\ x \equiv 2 \mod 7 \end{array}$ 

### HSC PLACEMENT TEST

Answer as many questions as you can. Reduce all fractions to lowest terms.

Name: \_\_\_\_\_Grade: \_\_\_\_\_Date: \_\_\_\_\_

Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a box around it.

- 1. A parallelogram is defined as a quadrilateral with two pairs of parallel sides. Using vectors, derive the following results:
  - (a) The opposite sides are congruent
  - (b) Diagonals bisect each other.

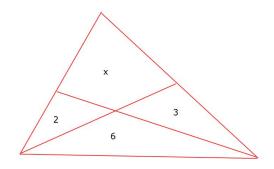
<sup>2.</sup> A repunit is a number consisting entirely of 1s. For example, 1, 11, 111, etc. are repunits. Prove that if n is not divisible by 2 or 5, then there is a repunit divisible by n.

3. The Fibonacci sequence is defined recursively via  $F_0 = 0, F_1 = 1, F_{n+2} = F_{n+1} + F_n$ . Prove that

 $F_1 + F_3 + \dots + F_{2n-1} = F_{2n}$ 

<sup>4.</sup> If the lengths of the bases of a trapezoid inscribed in a circle are 10 and 20, and the length of one of the legs is  $\sqrt{89}$ , find the length of a diagonal.

5. Find the area x.



6. Prove that for non-negative x, y, z, if  $xyz \ge 1$ , then  $(x+1)(y+1)(z+1) \ge 8$ .

7. How many 4-letter words can be formed such that each word has 4 different letters in increasing alphabetical order, such as AFNZ?

8. Factor completely:  $181^2 + 457^2 + 362 \cdot 457 - 362^2$ 

9. How many solutions does the equation x + y + z = 10 have in non-negative integers if  $x \le 3, y \le 6, z \le 9$ ?

10. Find the 3rd degree monic polynomial with roots r, s, t which satisfies

$$\frac{1}{r} + \frac{1}{s} + \frac{1}{t} = \frac{5}{8}$$
$$\frac{1}{rs} + \frac{1}{rt} + \frac{1}{st} = \frac{3}{8}$$
$$\frac{1}{rst} = \frac{1}{8}$$

#### COLLEGE BRIDGE PLACEMENT TEST

Answer as many questions as you can. Reduce all fractions to lowest terms.

Name: \_\_\_\_\_\_Grade: \_\_\_\_\_\_Date: \_\_\_\_\_

Write your answer and show your work in the space below each question. Clearly indicate your final answer by drawing a box around it.

1. By using the rearrangement inequality, or otherwise, prove that

$$a^4 + b^4 + c^4 + d^4 \ge 4abcd$$

if a, b, c and d are positive real numbers.

2. Let  $a_1, a_2, \cdots$  be a sequence defined by  $a_1 = \frac{1}{\sqrt{3}}, a_2 = \frac{1}{\sqrt{3}}$  and

$$a_{n+1} = \frac{a_n + a_{n-1}}{1 - a_n a_{n-1}}$$

for  $n \geq 2$ . Find  $a_n$  in terms of  $F_n$ , the *nth* Fibonacci number.

3. *n* is selected from the set  $\{1, 2, 3, \dots 100\}$  and the number  $f(n) = 2^n + 3^n + 5^n$  is formed. What is the total number of ways of selecting *n* so that f(n) is divisible by 4? 4. Compute  $(1 + \sqrt{3})^{100}$ .