

Ted Swartz of the Bronx Charter School for Better Learning and Arthur Powell of Rutgers spoke briefly about *Subordinating Teaching to Learning*, an approach to teaching that was developed by Caleb Gattegno (1911-1988) and is the basis both for the charter school and their professional work with teachers.

They suggested that there are many advanced level, upper grade topics in mathematics that can be explored and illustrated through the use of Cuisenaire rods, which are colored sticks that are multiples of centimeter cubes (see photo.)

Specifically, Dr. Swartz proposed that Cuisenaire rods could be used to explore the physical reality of: $(a+b)^2$.

After about fifteen minutes of rod exploration, during which participants built many gorgeous and intriguing constructions, Dr. Swartz asked participants to choose two differently colored (and sized) rods and create a square of each.

He then invited participants to make a “train” of the two lengths they had chosen (by putting them end to end with each other) and to create a square of the resultant length, or sum.

The challenge was to compare the sum of the areas of the two smaller squares (i.e., $a^2 + b^2$) with the area of the square of the sum of the two lengths (i.e., $(a+b)^2$).

After participants had constructed for themselves all three squares: a^2 , b^2 and $(a+b)^2$, they were challenged to place the two smaller ones on top of the larger one (as in a layer cake) in such a manner that the shapes of the uncovered space(s) might reveal how the areas of those remaining spaces were related to the original two lengths.

With much discussion, the conclusion was reached that the two spaces left uncovered were rectangles that could be described in terms of a and b : that is, a of the b 's or ab . There were two identical ab spaces, or $2ab$.

Thus it was concluded that $(a+b)^2 = a^2 + b^2 + 2ab$.

Dr. Swartz worked primarily with a visitor who was not a math teacher or math specialist, taking as much care as he could muster not to ask leading questions, ones that would risk getting the participant to answer correctly without her really understanding the question or the answer or the meaning of the task and leave her unable to proceed once he, the teacher, was off-site.

After the experiment with squaring the sum of two rods was concluded and discussed, the group then tackled building three cubes with the rods to test out the difference between the sum of two cubes and the cube of the sum of the two lengths (that is: What is the difference between $a^3 + b^3$ and $(a+b)^3$? The challenge was to describe the physical difference in terms of a and b .

That challenge engaged the personal attention of even the most experienced and talented mathematicians present, not to find the correct formula, but to construct and manipulate the cubes in a manner that revealed the nature of the relationships, in three dimensions.

Gattegno's approach, *Visible and Tangible Mathematics*, was experienced as an engaging and enlightening entry into a mathematics topic that all too many students experience as murky and dull.

