

Algebra

1. Let $C = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$. Find all matrices H such that $CH = 0$. When does $HC = 0$?
2. Find all matrices A with integer entries such that $A^2 = I$
3. Find a matrix A such that $A^3 = I$ and $A \neq I$.
4. Suppose $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$. Find all matrices B such that $AB = 0$.
5. Find a matrix that has no square root.
6. Show that the set of all matrices of the form $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ is closed under multiplication.
7. Find a matrix A such that $A^3 = 0$ but $A^2 \neq 0$.
8. Find all 2×2 matrices that commute with $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$. Generalize.

Geometry

1. Let $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ and $M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Compute MAM and describe it geometrically.
2. As above, find the inverses of MA and AM . What power of MA is the identity?
3. Find a matrix that orthogonally projects the plane onto the line $y = x$.
4. Let $D = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$. Show that D rotates the plane 45° counterclockwise. Find D^{-1} .
5. Describe the transformation DMD^{-1} in geometric terms, where D and M are defined as above.
6. Find a matrix that reflects the plane over the line $y = 2x$.
7. Find a 3×3 matrix that rotates space about the x-axis.