

The Level C class is the most advanced that we currently offer. You can read a description of this course on the summer program website. Additionally, we offer these sample problems to help illustrate the depth and breadth of material you will encounter at this level.

We don't expect you to solve all these problems; if you could, why would you take this class?

1. In triangle ABC , $AB = 13$, $BC = 15$, $CA = 14$, D is on line segment BC so that $BD = 5$, and E is the midpoint of AD . Find the area of triangle ABE .
2. All the circles in a chain of successively tangent circles of decreasing size are tangent to both sides of a 60 degree angle. If the length of the radius of the largest circle in this chain is 105, find the sum of the lengths of the radii of all the circles in the chain.
3. The function f is defined by $f(1) = 1$ and $f(n) = f(n - 1) + 2n$ for every positive integer n that is greater than or equal to 2. Write an equation expressing $f(n)$ explicitly in terms of n .
4. Find four prime numbers less than 100 which are factors of $3^{32} - 2^{32}$.
5. In the convex quadrilateral $ABCD$, points M and N lie on side AB such that $AM = MN = NB$, and the points P and Q lie on side CD such that $CP = PQ = QD$. Prove that $|AMCP| = |MNPQ| = (1/3)|ABCD|$. [Note that $|XYZ|$ denotes the area of region XYZ .]
6. The number 916238457 is an example of a nine-digit number which contains each of the digits exactly once. It also has the property that the digits 1 to 5 occur in their natural order, while the digits 1 to 6 do not. How many such numbers are there?
7. How many numbers are both multiples of 60^{60} and divisors of 120^{120} ?
8. Factor completely: $a^2 + 4b^2 - 9c^2 - 4ab$.
9. In parallelogram $ABCD$, $AB = 7$, $BC = 6$, and the length of diagonal AC is 6. Find the length of BD .
10. Find the volume of a regular tetrahedron that has a side whose length is 6.
11. If $x + x^{-1} = 7$, compute $x^8 + x^{-8}$.
12. Compute the units digit of $(2007 \times 2009 \times 2011)^n$, where n is the answer to this problem.
13. A circle is inscribed in a square whose sides have length 1. A second smaller circle is externally tangent to the first circle and tangent to two adjacent sides of the square. Find the radius of the smaller circle.
14. Find all ordered pairs (x, y) of positive integers that satisfy $4x^2 - y^2 = 315$.