Friday the thirteenth

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QUESTIONS

- *Does every year contain a Friday the thirteenth?*

- *What is the maximum number of Friday the thirteenths in a year?*

DAYS OF THE WEEK

- English: Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday

- French: lundi ,mardi, mercredi, jeudi, vendredi, samedi, dimanche

- Spanish: lunes, martes, miércoles, jueves, viernes, sábado, domingo

- German: Montag, Dienstag, Mittwoch, Donnerstag , Freitag , Samstag, Sonnstag

- Swahili: jumatatu ,jumanne, jumatano, alhamisi ,ijumaa ,jumamosi, jumapili

- Hungarian: hétfő , kedd, szerda, csütörtök , péntek, szombat , vasárnap

- Esperanto: lundo ,mardo, merkredo, ĵaŭdo, vendredo, sabato, dimanĉo

[Source: http://www.omniglot.com/language/time/days.htm]

- Mathematics: A, B, C, D, E, F, G

DAYS OF THE MONTH RHYME

Thirty days hath September,
April, June and November;
February has twenty eight alone
All the rest have thirty-one
Except in Leap Year, that's the time
When February's Days are twenty-nine

[Source: http://www.rhymes.org.uk/thirty_days_hath_september.htm]

NON-LEAP YEAR

MONTH	DAY OF THE 13TH	# OF DAYS >28
January	A	3
February	D	0
March	D	
April		
May		
June		
July		
August		
September		
October		
November		
December		

NON-LEAP YEAR

MONTH	DAY OF THE 13TH	# of days > 28
January	A	3
February	D	0
March	D	3
April	G	2
May	B	3
June	E	2
July	G	3
August	C	3
September	F	2
October	A	3
November	D	2
December	F	3

LEAP YEAR

MONTH	DAY OF THE 13TH	# OF DAYS >28
January	A	3
February	D	1
March		
April		
May		
June		
July		
August		
September		
October		
November		
December		

LEAP YEAR

MONTH	DAY OF THE 13TH	# OF DAYS >28
January	A	3
February	D	1
March	E	3
April	A	2
May	C	3
June	F	2
July	A	3
August	D	3
September	G	2
October	B	3
November	E	2
December	G	3

OBSERVATIONS

- **Every day of the week occurs at least once on the 13th of a month.**

- **No day occurs more than three times on the thirteenth of a month**

FREQUENCY OF
THE DAYS OF THE WEEK

DAY	NON –LEAP YEAR	LEAP YEAR
A	2	3
B	1	1
C	1	1
D	3	2
E	1	2
F	2	1
G	2	2

ADDITIONAL OBSERVATIONS

- In a non-leap year with three Friday the 13th's, they occur in February, March, and November.   [Day D]

- If a non-leap year has only one Friday the 13th, it occurs in May, June, or August.   [Day B, E, or C]

- In a leap year with three Friday the 13th's, they occur in January, April, and July, as in 2012.   [Day A]

- If a leap year has only one Friday the 13th, it occurs in May, June, or November.   [Day C, F, or E]

ADDITIONAL OBSERVATIONS

- In 2012, a leap year, January 13th was a Friday so A translated to Friday and the three Friday the 13th's were in January, April, and July.

- In 2013, January 13th will be on the 2012 December's day G + 3 which corresponds to 2012's day C. Since 2012's day A was a Friday, day C was Sunday and day F was Friday. In a non-leap year there are two F days that fall on the 13th. Thus, the two Friday the 13th's in 2013 will fall in September and December.

    Check a calendar!

EXTENSION

- *What is the greatest number of months that can elapse without a Friday the thirteenth?*

Hint: Extend the chart to two consecutive years. Each letter must appear at least twice. (Why?)

LEAP YEAR FOLLOWED BY NON-LEAP YEAR

MONTH	DAY OF THE 13TH	# OF DAYS >28
January	A	3
February	D	1
March	E	3
April	A	2
May	C	3
June	F	2
July	→ A ←	3
August	D	3
September	G	2
October	B	3
November	E	2
December	G	3

MONTH	DAY OF THE 13TH	# OF DAYS >28
January	C	3
February	F	0
March	F	3
April	B	2
May	D	3
June	G	2
July	B	3
August	E	3
September	→ A ←	2
October	C	3
November	F	2
December	A	3

NON-LEAP YEAR FOLLOWED BY NON-LEAP YEAR

MONTH	DAY OF THE 13TH	# OF DAYS >28
January	A	3
February	D	0
March	D	3
April	G	2
May	B	3
June	E	2
July	→ G ←	3
August	C	3
September	F	2
October	A	3
November	D	2
December	F	3

MONTH	DAY OF THE 13TH	# OF DAYS >28
January	B	3
February	E	0
March	E	3
April	A	2
May	C	3
June	F	2
July	A	3
August	D	3
September	→ G ←	2
October	B	3
November	E	2
December	G	3

NON-LEAP YEAR FOLLOWED BY LEAP YEAR

MONTH	DAY OF THE 13TH	# OF DAYS >28	MONTH	DAY OF THE 13TH	# OF DAYS >28
January	A	3	January	B	3
February	D	0	February	E	1
March	D	3	March	F	3
April	G	2	April	B	2
May	B	3	May	D	3
June	E	2	June	G	2
July	G	3	July	B	3
August	→ C ←	3	August	E	3
September	F	2	September	A	2
October	A	3	October	→ C ←	3
November	D	2	November	F	2
December	F	3	December	A	3

CONCLUSION

The longest possible stretch is 14 months.

- The 14 months stretch from July to September of the following year when the second year is not a leap year. According to Wikipedia, this happened or will happen in 2001 – 2002, 2012 – 2013, and 2018 – 2019.

- The 14 months stretch from August to October of the following year when the second year is a leap year. According to Wikipedia, this happened or will happen in 1999 – 2000 and 2027 – 2028.

TRIVIA TIDBIT

Each Gregorian 400-year cycle contains 146,097 days (365 × 400 = 146,000 normal days, plus 97 leap days). 146,097 ÷ 7 = 20,871 weeks. Thus, each cycle contains the same pattern of days of the week (and thus the same pattern of Fridays that are on the 13th).

DISTRIBUTION OF THE 13ᵗʰ DAY OVER 4800 MONTHS IN A GREGORIAN CYCLE

DAY OF THE WEEK	NUMBER OF OCCURENCES
Sunday	687
Monday	685
Tuesday	685
Wednesday	687
Thursday	684
Friday	→ → **688** ← ←
Saturday	684

Enjoy Friday the thirteenth this Friday!

And again in June!

-Sheila Krilov