FRIDAY THE THIRTEENTH

Questions:

- *Does every year contain a Friday the thirteenth?*

- *What is the maximum number of Friday the thirteenths in a year?*

- *What is the greatest number of months that can elapse without a Friday the thirteenth?*

To answer these questions, let's call the seven days of the week A, B, C, D, E, F, G

Month	Non-Leap Year followed by a Non-leap Year		Leap Year Followed by a Non-leap year	
	Day of the week of the 13th	# of days in the month beyond 28	Day of the week of the 13th	# of days in the month beyond 28
January	A	3	A	3
February	D	0	D	1
March	D	3	E	3
April	G	2	A	2
May	B	3	C	3
June	E	2	F	2
July	G	3	A	3
August	C	3	D	3
September	F	2	G	2
October	A	3	B	3
November	D	2	E	2
December	F	3	G	3

Observe that:

- Every day of the week occurs at least once on the 13th of a month.

- No day occurs more than three times on the 13th of a month.

FREQUENCY OF DAYS OF THE WEEK

	A	B	C	D	E	F	G
Non-Leap Year	2	1	1	3	1	2	2
Leap year	3	1	1	2	2	1	2

Additional observations:

- In a non-leap year with three Friday the 13th's, they occur in February, March, and November.   {Day D]

- If a non-leap year has only one Friday the 13th, it occurs in May, June, or August. [Day B, E, or C]

- In a leap year with three Friday the 13th's, they occur in January, April, and July, as in 2012.   [Day A]

- If a leap year has only one Friday the 13th, it occurs in May, June, or November. [Day C, F, or E]

- In 2012, a leap year, January 13th was a Friday so A translated to Friday and the three Friday the 13th's were in January, April, and July.

- In 2013, January 13th will be on December's day G + 3 which corresponds to C. Since A corresponds to Friday, C corresponds to Sunday. Also F corresponds to Friday. In a non-leap year there are two F days that fall on the 13th. The two Friday the 13th's in 2013 will fall in September and December. Check a calendar!

Extension:
- *What is the greatest number of months that can elapse without a Friday the 13th*

Hint: Extend the chart to two consecutive years. Each letter must appear at least twice. (Why?)   There are three possible cases.

Month	Non-Leap Year followed by Non-Leap Year		Non-leap Year followed by Leap Year		Leap Year followed by Non-Leap Year	
	Day of the week of Friday the 13th	# of days in the month beyond 28	Day of the week of Friday the 13th	# of days in the month beyond 28	Day of the week of Friday the 13th	# of days in the month beyond 28
January	A	3	A	3	A	3
February	D	0	D	0	D	1
March	D	3	D	3	E	3
April	G	2	G	2	A	2
May	B	3	B	3	C	3
June	E	2	E	2	F	2
July	G	3	G	3	A	3
August	C	3	C	3	D	3
September	F	2	F	2	G	2
October	A	3	A	3	B	3
November	D	2	D	2	E	2
December	F	3	F	3	G	3
January	B	3	B	3	C	3
February	E	0	E	1	F	0
March	E	3	F	3	F	3
April	A	2	B	2	B	2
May	C	3	D	3	D	3
June	F	2	G	2	G	2
July	A	3	B	3	B	3
August	D	3	E	3	E	3
September	G	2	A	2	A	2
October	B	3	C	3	C	3
November	E	2	F	2	F	2
December	G	3	A	3	A	3

The longest possible stretch is 14 months.:

- The 14 months stretch from July to September when the following year is not a leap year. According to Wikipedia, this happened or will happen in 2001 – 2002, 2012 – 2013, and 2018 – 2019.

- The 14 months stretch from August to October when the following year is a leap year. According to Wikipedia, this happened or will happen in 1999 – 2000 and 2027 – 2028.

Trivia tidbit (from Wikipedia):

Each Gregorian 400-year cycle contains 146,097 days (365 × 400 = 146,000 normal days, plus 97 leap days). 146,097 ÷ 7 = 20,871 weeks. Thus, each cycle contains the same pattern of days of the week (and thus the same pattern of Fridays that are on the 13th). The 13th day of the month is slightly more likely to be a Friday than any other day of the week. On average, there is a Friday the 13th once every 212.35 days (compared to Thursday the 13th, which occurs only once every 213.59 days).

The distribution of the 13th day over the 4,800 months is as follows:

Day of the week	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of occurrences	687	685	685	687	684	**688**	684