Comments/Additional Problems

In **Example 1** the answer, $\pi/4$, is independent of the size of the square. In fact, if you have a similar configuration with n^2 circles inside a square, $n \geq 1$, the probability is still $\pi/4$. [Thanks to Fred Galli for this last observation.]

You can generalize **Example 2** in various ways. For instance, if A and B are selected randomly from the real number line, then the probability of real roots is 1: Consider $A, B \in [-\mathrm{N}, \mathrm{N}]$ and get the probability $1 - \frac{2}{3\sqrt{N}}$. Now let $N \to \infty$. However, if we consider the quadratic equation $ax^2 + bx + c = 0$, where *a*, *b*, *c* are randomly selected real numbers, then the probability of real roots is

$$\frac{41 + 3\ln 4}{72} \approx 0.6272.$$

In **Example 3**, you can ask how long the friends should be willing to wait to guarantee that the probability of their meeting is greater than $1/2$. [Thanks to Sheila Krilov for the suggestion.]

In the **Buffon Needle Problem**, it can be shown that if $\ell > d$, then the probability of a crossing is

$$\frac{\pi d - 2\left(d \sin^{-1}\left(\frac{d}{\ell}\right) + \sqrt{\ell^2 - d^2} - \ell \right)}{\pi d}.$$

For additional information on **Bertrand's Paradox,** see http://en.wikipedia.org/wiki/Bertrand%27s_paradox_%28probability%29 or http://www.cut-the-knot.org/bertrand.shtml**.**

Additional Problems:

Suppose two numbers, *x* and *y*, are generated at random, where $0 < x < 3$ and $0 < y < 6$. What is the probability that the sum of these numbers is less than or equal to 2? **[1/9]**

Suppose two numbers, *x* and *y*, are generated at random, where $0 < x < 1$ and $0 < y < 1$. What is the probability that the product of the two numbers is less than $\frac{1}{2}$? $\left[\mathbf{\frac{1}{2} - \frac{1}{2} \ln\left(\frac{1}{2}\right) \approx 0.8466} \right]$

Two friends, Devon and Julie, are shopping at the mall. They agree to split up for a time and then meet for lunch. They plan to meet in front of Neiman Marcus between 12:00 noon and 1:00 P M. The one who arrives first agrees to wait 15 minutes for the other to arrive. After 15 minutes, that person will leave and continue shopping. What is the probability that they will meet if each of the friends arrives at any time between 12:00 noon and 1:00 P M? **[7/16]**

In the answer to the Buffon Needle Problem, do the positions of the quantities d and l seem reasonable? Why do you think π appears in the answer?

A darts player lands a dart somewhere in the first quadrant. What is the probability that the dart lies below the curve $y = e^x$?
[**Hint**: Assume that the dart lies within a square of size *N* with sides along the coordinate axes and one vertex at the origin. Then see what happens as $N \to \infty$.]