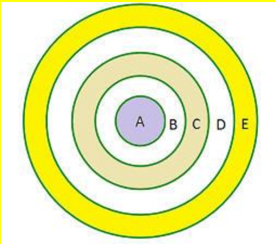
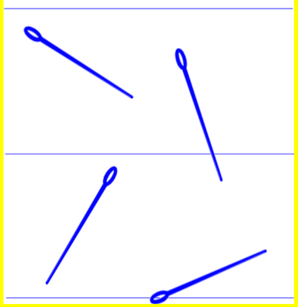


Geometric Probability: An Appetizer

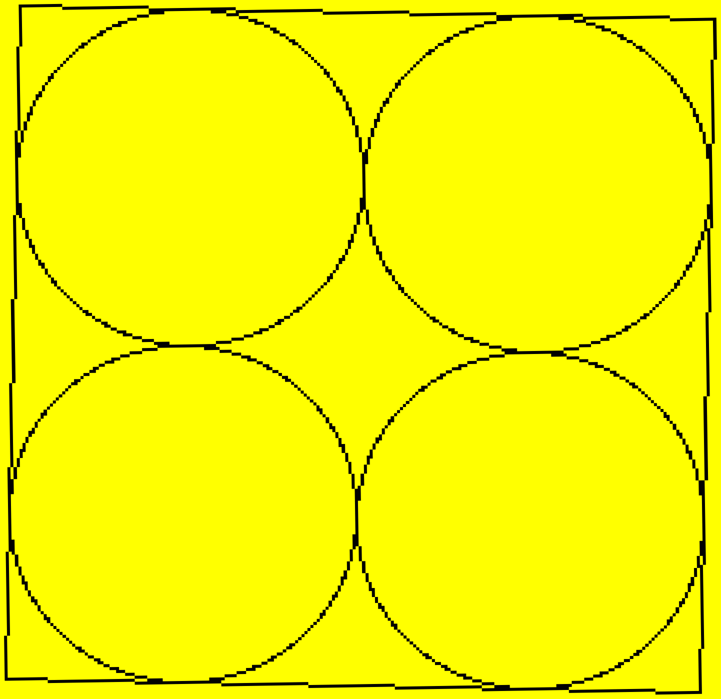


Dr. Henry Ricardo **New York Math Circle**

October 28, 2013

Example 1

What is the probability of a dart's landing in a circular region?



Example 2

Consider the equation $x^2 + Ax + B = 0$. Let constants A and B be chosen independently with the uniform distribution on the interval [0,1]. Calculate the probability that the equation has real roots.

Example 3

Two friends who take the subway to work from the same station arrive at the station uniformly randomly between 7 and 7:20 in the morning. They are willing to wait for one another for 5 minutes, after which they take a train whether together or alone. What is the probability of their meeting at the station?

The Buffon Needle Problem

Parallel lines, *d* units apart, are ruled on a plane surface. A needle of length $\ell < d$ units is thrown at random onto the plane. What is the probability that it will intersect one of the parallel lines?

—Georges Louis Leclerc, Comte de Buffon
(1733: Paris Academy of Sciences; 1777: *Essai d'arithmétique morale*)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BUFFON
1707 - 1788

Answer

$$p = \frac{2l}{\pi d}$$

Some Generalizations

‘Needle’

– length $\ell \geq d$

– shape: circle, ellipse, regular polygon, wet noodle

‘Plane Surface’

– rectangular tiles, hexagonal tiles

– a collection of n lines through a single point with uniform angular spacing between the lines

The University of Cambridge

The Broken Stick Problem

A rod is marked at random at two points, and then divided into three parts at these points; shew [sic] that the probability of its being possible to form a triangle with the pieces is $\frac{1}{4}$.

—Senate-House Examinations, Cambridge University, January 18, 1854

A Generalization

Break the stick into n pieces and find the probability that no one of the pieces exceeds the sum of the rest, so that a polygon of n or fewer angles can be made of the n pieces.

Answer: $$p = 1 - \frac{n}{2^{n-1}}$$

Bertrand's Paradox

Consider an equilateral triangle inscribed in a circle. Suppose a chord of the circle is chosen at random. What is the probability that the chord is longer than a side of the triangle?

—Joseph Bertrand (1889: *Calcul des probabilités*)

Answers (!)

The answer depends on the method of choosing a chord "at random":

- Choose two random points on the circumference of the circle and draw the chord joining them. . . Answer is 1/3

- Choose a radius of the circle, choose a point on the radius and construct the chord through this point and perpendicular to the radius. . . Answer is 1/2

- Choose a point anywhere within the circle and construct a chord with the chosen point as its midpoint. . . Answer is 1/4