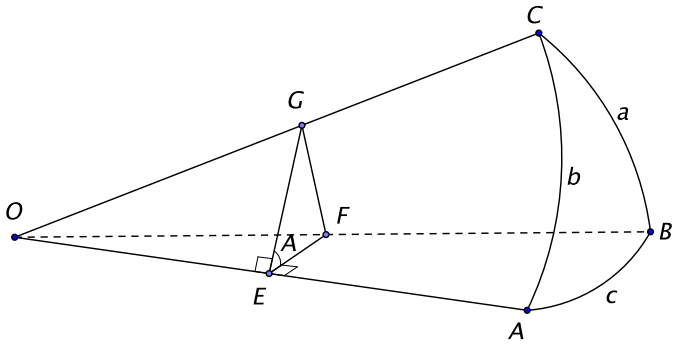
**Problems**

1. Imagine a perfectly spherical Earth, and two concentric rings around the Earth's equator. The first ring fits tightly around the equator, and the second ring is 1 meter longer in circumference (i.e. about 0.0000025% longer). How high is the second ring above the first? What does your intuition say?
2. If you fly a plane making sure that at every moment you are heading northeast, where will your path take you?
3. An explorer walks one mile due south, turns and walks one mile due east, turns again and walks one mile due north. He finds himself back where he started. Where on Earth did he start? There is more than one solution: can you find all?
4. Explain why it is colder in the winter than in the summer. (Note that the Earth is closest to the Sun in January.)
5. If you know the origins of some units of distance, you'll know the measurements of the Earth.
   (a) A *nautical mile* is 1 minute of latitude measured along any meridian (that is, 1/60 of a degree). What's the radius of the Earth in nautical miles? (Assume a spherical Earth; in reality, because of flattening, the Earth's equatorial and polar radii differ slightly -- by about 0.3%.)
   (b) A *meter* was originally defined as one ten-millionth of the length of the meridian from the equator to the North Pole through Paris. (It differs by less than 0.02% from the modern definition, which in terms of the speed of light.) Given this information, what is the radius of the Earth in kilometers?
   (c) How long is a nautical mile in meters? (The answer, rounded to the nearest meter, is the current official definition of a nautical mile.)
6. Prove that every cross-section of the sphere by a plane is a circle. (*Hint:* Use Pythagorean Theorem.)
7. Prove that the altitude of the North Star above the horizon is equal to the latitude of the observer. *[Van Brummelen 2013, p. 39 #3]*
8. How many miles is 1° of longitude at New York City (lat. 40.8° N)? *[Van Brummelen 2013, p. 40 #8, attributed to Raymond N. Greenwell]*
9. Find, correct to the *nearest 10 nautical miles*, the distance between New York City (40° 49' N, 73° 58' W) and Gibraltar (36° 06' N, 5° 21' W). *[NYS Regents, Jun. 1944, Trigonometry #29]*

   (*Hint:* Use the Great-circle distance formula, or the spherical Law of Cosines on the triangle formed by the two cities and the North Pole -- which measure of this triangle is determined by the longitudes?)
10. Develop an alternate proof of the spherical Law of Cosines using the diagram below where angles $\angle OEF$ and $\angle OEG$ are right by construction. (Here, *a, b,* and *c* are angle measures of the arcs.)




*Hint:* Apply the planar Law of Cosines to *ΔOGF* and *ΔGFE* to solve for their common side and combine the two statements. Simplify the new statement using the Pythagorean theorem on *OFE* and *OEG*. Then solve for cos *a*.

*[Moritz 1913, 38-39 #5, quoted in Van Brummelen 2013, p.107 #10.]*