Worksheet on coin flipping: sequences of heads and tails

Here are two sequences of H's and T's. One of them was made by a random number generator on the internet. I made the other one myself, just by writing a string of H's and T's longhand on a piece of paper, trying not to put them in any pattern.

Sequence #1:

T, H, T, H, T, T, H, T, T, H, T, H, H, H, H, H, H, H, H, T, H, T, T,
T, T, H, T, T, H, H, T, T, H, T, H, H, H, H, T, T, H, H, H, H, T, H,
T, T, T, T, H, H, T, H, H, H, H, T, H, T, T, T, H, T, H, H, T, H, H,
T, H, T, H, H, T, H, T, H, T, H, T, T, H, T, H, H, T, H, H, H, T, T, H,
H, H, H, T, T, H, H, T

Sequence #2:

H, T, T, H, T, H, H, T, H, T, H, H, T, T, H, H, T, T, H, T, T, H, H,
T, T, H, H, T, T, H, H, T, T, H, T, H, H, T, T, T, T, H, H, T, T,
H, H, T, H, T, H, T, H, H, T, H, T, H, T, T, H, T, H, H, T, H, H, H, T,
H, T, H, H, T, H, T, H, T, H, H, T, T, H, H, T, T, H, H, T, T, H, T,
T, H, H, H, T, H, T, H

Can we figure out which sequence is which?

For reference, Sequence # 1 has 55 heads and 45 tails. Sequence # 2 has 51 heads and 49 tails.

On coin flipping

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Thoughts on the first worksheet.

One difference between the two sequences is that the first sequence has more streaks than the second one. Let's say that a streak is a sequence which is either all H's or all T's. The first sequence has a streak of 8 H's in a row. In addition, it has four other streaks of length 4. The second sequence has one streak of length 5, and no other streaks of length at least 4. That's a pretty striking difference. But which behavior do we expect from a genuine sequence of coin flips?

Let's look at something related. We can count how many times HHH appears in the sequence and how many times HHT appears in the sequence. Sequence one has 14 HHH's and 13 HHT's. Sequence two has 1 HHH and 17 HHT's. Again we have a pretty striking difference between the two sequences. As we discussed in class, if we flip a coin three times, it's equally likely to turn out HHH or HHT. So the behavior of sequence one looks reasonable for a coin, and the behavior of sequence two looks suspicious. You can try a similar analysis with TTT and TTH. The results are similar, although not as dramatic.

Now I confess that the second sequence is the one that I made up. I got the first sequence from an online random number generator (you can find it at http://www.randomizer.org/form.htm). Most people trying to write a random string of H's and T's don't put enough streaks in their sequences. After I wrote HH, I was much more likely to follow that with a T than with an H. A real coin would be equally likely to land H or T, regardless of how it landed in the previous flips. I think that after writing a couple of H's, I began to feel nervous about writing too many H's in a row, and I tended to switch to T.

Further sequences

Here are some other sequences of H's and T's. Some come from the random number generator. Some of them are versions that I wrote trying to impersonate a coin. It might be interesting to try to figure out which is which. I made one sequence where I tried to learn from my experience by looking back at what I had written and adding a bunch of streaks.

Sequence # 3

1

T, T, H, H, H, T, T, T, H, H, H, T, H, H, T, T, H, H, T, T, H, T, T,
H, H, H, T, T, H, T, H, H, T, T, H, H, H, T, T, H, T, H, T, T, H, H,
T, H, T, H, H, T, H, T, T, H, H, T, H, T, T, H, H, T, T, H, H, T, T,
H, T, T, H, H, T, H, T, H, H, T, H, H, T, T, H, H, H, T, T, H, H, H,
T, T, H, T, H, T, T, H

Sequence # 4

H, T, T, H, T, H, H, T, H, T, H, T, T, H, T, H, H, T, H, T, T, T, T,
T, T, T, H, H, T, T, T, H, T, H, T, H, H, T, H, T, H, H, H, H, H, T,
H, H, T, H, H, T, T, T, H, H, T, T, H, H, H, T, T, H, H, H, H, H, T,
T, H, H, T, T, H, T, H, H, T, T, T, T, H, H, T, H, T, T, H, H, H,
T, T, H, H, H, H, T, H

Sequence # 5

T, T, T, H, H, H, T, T, T, T, H, H, T, H, T, H, H, T, T, T, T, H, T,
T, T, H, H, T, H, T, H, T, H, H, T, H, H, T, T, T, H, H, H, H, T, H, T,
T, T, H, H, T, T, T, H, T, T, T, H, T, H, H, H, T, H, H, H, T, T, H, H,
T, H, H, H, T, T, H, T, T, T, H, T, H, H, H, H, T, T, T, T, H, T, H,
T, H, H, H, T, T, T, T

Sequence # 6

H, T, T, T, H, H, H, T, H, H, H, T, H, H, T, H, H, T, T, T, H, T, H,
T, H, H, T, H, H, H, T, H, T, T, T, H, T, T, H, H, H, H, H, T, T, H,
H, H, H, H, T, T, H, T, T, H, T, H, H, H, H, H, H, H, T, H, H, H, T,
T, H, H, T, T, H, T, H, T, T, H, H, T, T, T, T, T, T, T, T, T, H, T,
H, T, T, T, H, H, H, T

Further questions

Here are a couple other questions related to our discussion in different directions.

1. Suppose that I flip a coin 100 times. Which is more likely: to get exactly 70 heads, or to get 71-100 heads?

2. Suppose a colleague approaches you with a coin and says, "this coin is biased. It comes up heads 51 % of the time, and tails 49 % of the time." The colleague goes on: "I did an experiment. I flipped the coin 1000 times. It came up heads 511 times." Do you find this evidence convincing? If not, how many times would I need to flip the coin to get enough evidence to test the hypothesis that the coin comes up heads 51 % of the time?